David R. Merkel

Oct 16/48

ELEMENTS

OF

DRAWING AND MENSURATION

APPLIED TO

THE MECHANIC ARTS.

BY CHARLES DAVIES, LL.D.,

AUTHOR OF FIRST LESSONS IN ARITHMETIC, ARITHMETIC, ELEMENTARY ALGEBRA, ELEMENTS OF SURVEYING, ELEMENTS OF DESCRIPTIVE GEOMETRY, SHADES, SHADOWS, AND PERSPECTIVE, ANALYTICAL GEOMETRY, DIFFERENTIAL AND INTEGRAL CALCULUS.

NEW YORK:

PUBLISHED BY A. S. BARNES & CO.,

No. 51 JOHN STREET.

1846.

Stereotyped by
RICHARD C. VALENTINE,
New York.

C. A. ALVORD, Printer,
Corner of John and Dutch streets.

PREFACE.

The design of the present work is to afford an elementary text-book of a practical character, adapted to the wants of a community, where every day new demands arise for the applications of science to the useful arts. There is little to be done, in such an undertaking, except to collect, arrange, and simplify, and to adapt the work, in all its parts, to the precise place which it is intended to fill.

The introduction into our schools, within the last few years, of the subjects of Natural Philosophy, Astronomy, Mineralogy, Chemistry, and Drawing, has given rise to a higher grade of elementary studies ; and the extended applications of the mechanic arts call for additional information among practical men.

To understand the most elementary treatise on Natural Philosophy, or the simplest work on the Mechanic Arts, or even to make a plane drawing, some knowledge of the principles of Geometry is indispensable ; and yet, those in whose hands such works are generally placed, or who are called upon to make plans in the mechanic arts, feel that they have hardly time to go through with a full course of exact demonstration.

The system of Geometry is a connected chain of rigorous logic. Every attempt to compress the reasoning, by abridging it at the expense of accuracy, has been uniformly and strongly condemned.

All the truths of Geometry necessary to carry out fully the plan of the present work, are made accessible to the general reader, without departing from the exactness of the geometrical methods. This has been done by omitting the demonstrations altogether, and relying for the impression of each particular truth on a pointed question and an illustration by a diagram. In this way, it is believed that all the important properties of the geometrical figures may be learned in a few weeks; and after these properties are developed in their practical applications, the mind receives a conviction of their truth little short of what is afforded by rigorous demonstration.

The work is divided into seven Books, and each book is subdivided into sections.

In Book I. the properties of the geometrical figures are explained by questions and illustrations.

In Book II. are explained the construction and uses of the various scales, and also the construction of geometrical figures. It is, as its title imports, Practical Geometry.

Book III. treats of Drawing.—Section I., of the Ele ments of the Art; Section II., of Topographical Drawing; and Section III., of Plan-Drawing.

Book IV. treats of Architecture,—explaining the different orders, both by descriptions and drawings.

Book V. contains the application of the principles of Geometry to the mensuration of surfaces and solids. A separate rule is given for each case, and the whole is illustrated by numerous and appropriate examples.

Book VI. is the application of the preceding parts to Artificers' Work. It contains full explanations of all the scales and measures used by mechanics—the construction of these scales—the uses to which they are applied

—and specific rules for the calculations and computations which are necessary in practical operations.

Book VII. is an introduction to Mechanics. It explains the nature and properties of matter, the laws of motion and equilibrium, and the principles of all the simple machines.

From the above explanations, it will be seen that the work is entirely practical in its objects and character Many of the examples have been selected from a small work somewhat similar in its object, recently published in Dublin, by the Commissioners of National Education, and some from a small French work of a similar character. Others have been taken from Bonnycastle's Mensuration, and the Library of Useful Knowledge was freely consulted in the preparation of Book VII. A friend, Lt. Richard Smith, also furnished most of the first and second sections of Book III.; and the third section was chiefly taken from an English work.

The author has indulged the hope that the present work, together with his First Lessons in Arithmetic for Beginners, his Arithmetic, Elementary Algebra, and Elementary Geometry, will form an elementary course of mathematical instruction adapted to the wants of Practical men, Academies and the higher grade of schools.

West Point, *March*, 1846.

DAVIES'

COURSE OF MATHEMATICS.

DAVIES' FIRST LESSONS IN ARITHMETIC—For beginners.

DAVIES' ARITHMETIC.—Designed for the use of Academies and Schools.

KEY TO DAVIES' ARITHMETIC

DAVIES' UNIVERSITY ARITHMETIC—Embracing the Science of Numbers, and their numerous applications.

KEY TO DAVIES' UNIVERSITY ARITHMETIC

DAVIES' ELEMENTARY ALGEBRA—Being an Introduction to the Science, and forming a connecting link between ARITHMETIC and ALGEBRA.

KEY TO DAVIES' ELEMENTARY ALGEBRA.

DAVIES' ELEMENTARY GEOMETRY.—This work embraces the elementary principles of Geometry. The reasoning is plain and concise, but at the same time strictly rigorous.

DAVIES' ELEMENTS OF DRAWING AND MENSURATION —Applied to the Mechanic Arts.

DAVIES' BOURDON'S ALGEBRA—Including Sturms' Theorem,— Being an Abridgment of the work of M. Bourdon, with the addition of practical examples.

DAVIES' LEGENDRE'S GEOMETRY AND TRIGONOMETRY. —Being an Abridgment of the work of M. Legendre, with the addition of a Treatise on MENSURATION OF PLANES AND SOLIDS, and a Table of LOGARITHMS and LOGARITHMIC SINES.

DAVIES' SURVEYING—With a description and plates of the THEODOLITE, COMPASS, PLANE-TABLE, and LEVEL: also, Maps of the TOPOGRAPHICAL SIGNS adopted by the Engineer Department—an explanation of the method of surveying the Public Lands, and an Elementary Treatise on NAVIGATION.

DAVIES' ANALYTICAL GEOMETRY—Embracing the EQUATIONS OF THE POINT AND STRAIGHT LINE—of the CONIC SECTIONS—of the LINE AND PLANE IN SPACE—also, the discussion of the GENERAL EQUATION of the second degree, and of SURFACES of the second order.

DAVIES' DESCRIPTIVE GEOMETRY,—With its application to SPHERICAL PROJECTIONS.

DAVIES' SHADOWS AND LINEAR PERSPECTIVE.

DAVIES' DIFFERENTIAL AND INTEGRAL CALCULUS.

CONTENTS.

BOOK I.—SECTION I.

SECTION II.

SECTION III.

BOOK II.—SECTION I.

BOOK III.—SECTION I.

SECTION II.

SECTION III.

BOOK IV.—SECTION I.

SECTION II.

BOOK V.—SECTION I.

SECTION II.

BOOK VI.

SECTION II.

SECTION III.

SECTION IV.

GEOMETRY.

BOOK I.

SECTION I.

OF LINES AND ANGLES.

1. *What is a line?*

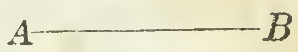
A *Line* is length, without breadth or thickness.

2. *What are the extremities of a line called?*

The *Extremities of a Line* are called points; and any place between the extremities, is also called a point.

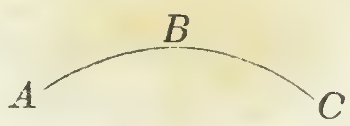
3. *What is a straight line?*

A *Straight Line*, is the shortest distance from one point to another. Thus, AB is a straight line, and the shortest distance from A to B.



4. *What is a curve line?*

A *Curve Line*, is one which changes its direction at every point. Thus, ABC is a curve line.



5. *What does the word line mean?*

The word *Line*, when used by itself, means a straight line; and the word *Curve*, means a curve line.

6. *What is a surface?*

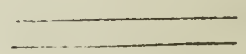
A *Surface* is that which has length and breadth, without height or thickness.

7. *What is a plane*, or *plane surface?*

A *Plane* is that which lies even throughout its whole extent, and with which a straight line, laid in any direction, will exactly coincide.

8. *When are lines said to be parallel?*

Two straight lines are said to be parallel when they are at the same distance from each other at every point. Parallel lines will never meet each other.

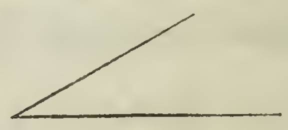


9. *When are two curves said to be parallel?*

Two curves are said to be parallel or *concentric*, when they are at the same distance from each other. Parallel curves will not meet each other.

10. *What are oblique lines?*

Oblique lines are those which approach each other, and meet if sufficiently prolonged.



11. *What are horizontal lines?*

Lines which are parallel to the horizon, or to the water level, are called *Horizontal Lines.* Thus, the eaves of a house are horizontal.

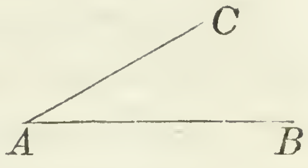
12. *What are vertical lines?*

All plumb lines are called *Vertical Lines.* Thus, trees and plants grow in vertical lines.

13. *What is an angle? How is it read?*

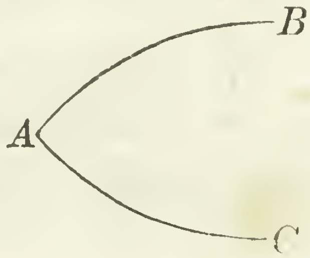
An *Angle* is the opening or inclination of two lines which

meet each other in a point. Thus the lines AC, AB, form an angle at the point A. The lines AC, and AB, are called the sides of the angle, and their intersection A, the *vertex*.

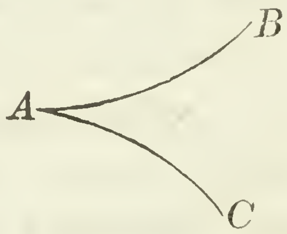


The angle is generally read by placing the letter at the vertex in the middle: thus, we say the angle CAB. We may, however, say simply, the angle A.

14. *May angles be formed by curved lines?*

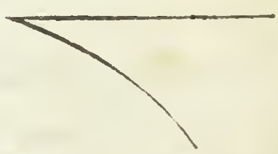


Yes, either by two curves, CA, BA forming the angle A, called a *curvilinear angle:*



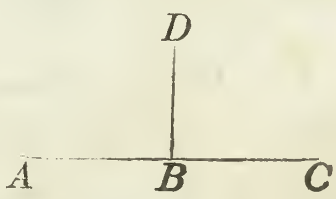
Or, by two curves AC, AB, forming the angle A:

Or, by a straight line and curve, which is called a *mixtilinear angle.*



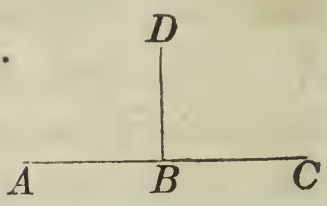
15. *When is one line said to be perpendicular to another?*

One line is perpendicular to another, when it inclines no more to the one side than to the other. Thus, the line DB is perpendicular to AC, and the angle DBA is equal to DBC.



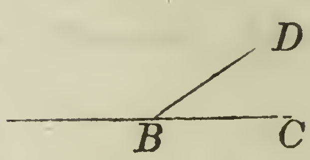
16. *What are the angles called?*

When two lines are perpendicular to each other, the angles which they form are called right angles. Thus, *DBA* and *DBC* are right angles. Hence, all right angles are equal to each other.



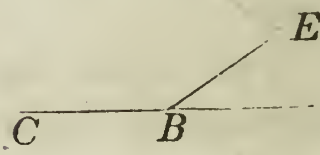
17. *What is an acute angle?*

An acute angle is less than a right angle. Thus, *DBC* is an acute angle.



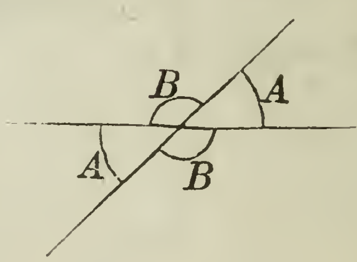
18. *What is an obtuse angle?*

An obtuse angle is greater than a right angle. Thus, *EBC* is an obtuse angle.



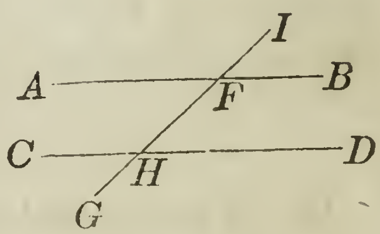
19. *If two lines intersect each other, what follows?*

If two lines intersect each other, the opposite angles *A* and *A* are called *vertical angles.* These angles are equal to each other, and so also are the opposite angles *B* and *B*.



20. *What follows when two parallel lines are cut by a third line?*

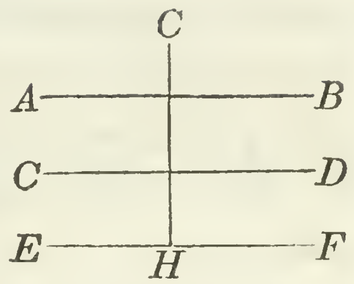
If two parallel lines *CD*, *AB*, are cut by a third line *IG*, the angles *IHD* and *AFG*, are called *alternate* angles. These angles are equal to each other. The angle *IHD* is also equal to the angle *IFB*, and to the opposite angle *CHG*.



21. *What follows when a line is perpendicular to one of several parallel lines?*

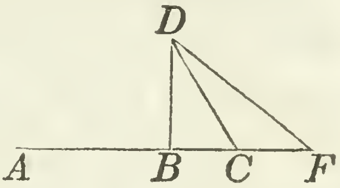
If a line be perpendicular to one of several parallel

lines, it will be perpendicular to all the others. Thus, if AB, CD, and EF, be parallel, the line CH drawn perpendicular to AB, will also be perpendicular to CD and EF.



22. *How many lines can be drawn from one point perpendicular to a line?*

From the same point D, only one line DB, can be drawn, which will be perpendicular to AB.



23. *If oblique lines are also drawn, what follows?*

If oblique lines be drawn, as DC, DF, then:—

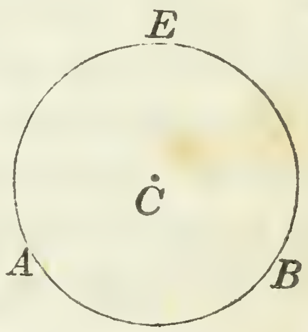
1st. The perpendicular DB, will be shorter than any of the oblique lines.

2d. The oblique lines which are nearest the perpendicular, will be less than those which are more remote.

OF THE CIRCLE AND MEASUREMENT OF ANGLES.

24. *What is the circumference of a circle?*

The circumference of a circle is a curve line, all the points of which are equally distant from a certain point within, called the centre. Thus, if all the points of the curve AEB are equally distant from the centre C, this curve will be the circumference of a circle.



25. *For what is the circumference of a circle used?*

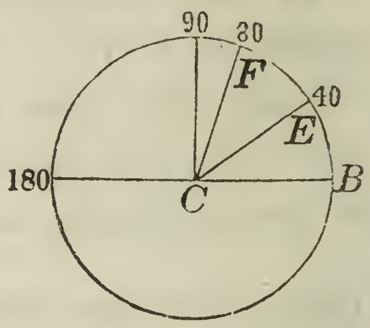
The circumference of a circle is used for the measurement of angles.

26. *How is it divided?*

It is divided into 360 equal parts called degrees, each

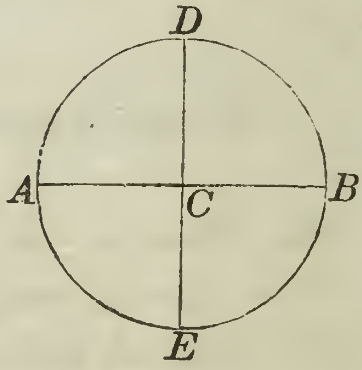
degree is divided into 60 equal parts called minutes, and each minute into 60 equal parts called seconds. The degrees, minutes, and seconds, are marked thus, °, ′, ″; and 9° 18′ 10″, are read, 9 degrees, 18 minutes, and 10 seconds.

27. *How are the angles measured?*



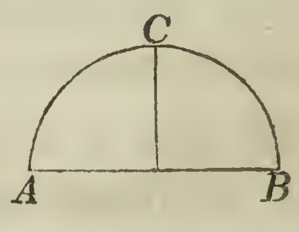
Suppose the circumference of a circle to be divided into 360 equal parts, beginning at the point *B*. If, through the point of division marked 40, we draw *CE*, then, the angle *ECB* will be equal to 40 degrees. If we draw *CF* through the point of division marked 80, it will make *CB* an angle equal to 80 degrees.

28. *How many degrees are there in one right angle,—in two—in three—in four?*



If two lines *AB*, *DE*, are perpendicular to each other, the four angles *BCD*, *DCA*, *ACE*, and *ECB*, will be equal. These two lines will divide the circumference of the circle into the four equal parts *BD*, *DA*, *AE*, and *EB*, and each part will measure one of the right angles. But one quarter of 360 degrees, is 90 degrees. Hence, one right angle contains 90 degrees, two right angles 180 degrees, three right angles 270 degrees, and four right angles 360 degrees.

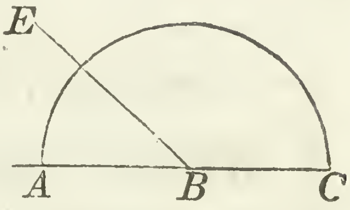
29. *What is one quarter of the circumference called?—one half of it?*



One quarter of the circumference is called a *quadrant*, and contains 90 degrees. One half of the circumference is called a *semi-circumference*, and con-

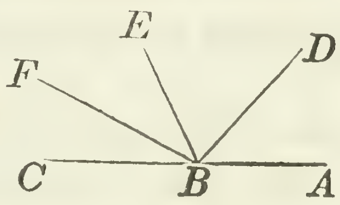
tains 180 degrees. Thus, AC is a quadrant, and ACB is a semi-circumference.

30. *If one straight line meets another, what is the sum of the two angles equal to?*



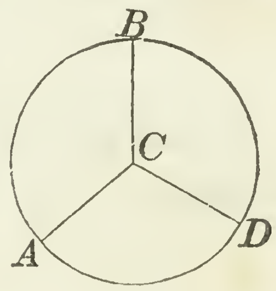
If a straight line EB meets another straight line AC, the sum of the angles ABE and EBC, will be equal to two right angles, since these two angles are measured by half the circumference.

31. *If there be several angles, what will their sum be equal to?*



If there be several angles CBF, FBE, EBD, DBA, formed on the same side of a line, their sum, for a like reason, will be equal to two right angles.

32. *What is the sum of all the angles formed about a point equal to?*



The sum of all the angles ACB, BCD, DCA, which can be formed about any point as C, is equal to four right angles, or 360 degrees, since they are measured by the entire circumference.

SECTION II.

OF PLANE FIGURES.

1. *What is a plane figure?*

A *plane* figure is a portion of a plane, terminated on all sides by lines, either straight or curved.

2. *When the bounding lines are straight, what is it called?*

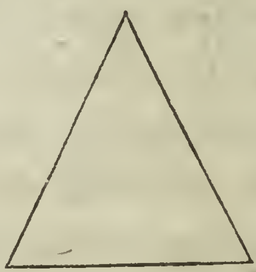
If the bounding lines are straight, the space they enclose is called a *rectilineal* figure, or polygon.

3. *What are the lines themselves called?*

The lines themselves, taken together, are called the *perimeter* of the polygon. Hence, the perimeter of a polygon is the sum of all its sides.

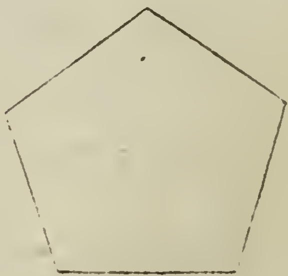
4. *Name the different kinds of polygons.*

A polygon of three sides, is called a triangle.

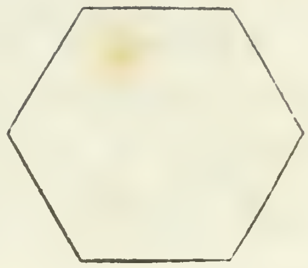


A polygon of four sides, is called a quadrilateral.

A polygon of five sides, is called a pentagon.



A polygon of six sides, is called a hexagon.



A polygon of seven sides, is called a heptagon.

A polygon of eight sides, is called an octagon.

A polygon of nine sides, is called a nonagon.

A polygon of ten sides, is called a decagon.

A polygon of twelve sides, is called a dodecagon.

5. *What is the perimeter of a polygon?*

The perimeter of a polygon is the sum of all its sides.

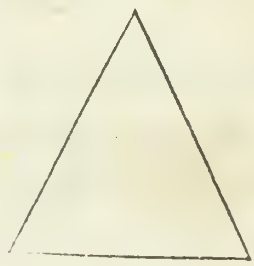
6. *What is the least number of straight lines which can enclose a space?*

Three straight lines, are the smallest number which can enclose a space.

7. *Name the several kinds of triangles.*

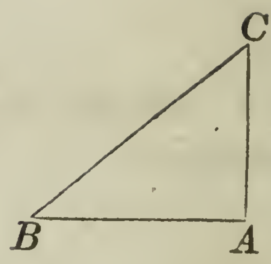
First.—An equilateral triangle, which has its three sides all equal.

Second.—An isosceles triangle, which has two of its sides equal.



Third.—A scalene triangle, which has its three sides all unequal.

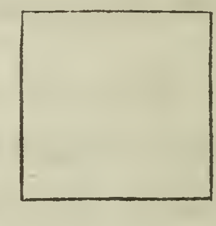
Fourth.—A right-angled triangle, which has one right angle. In the right-angled triangle *BAC*, the side *BC* opposite the right angle, is called the hypothenuse.



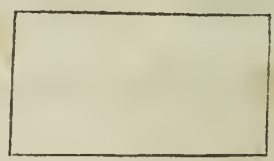
8. *What is the base of a triangle?—what its altitude?* The *base* of a triangle is the side on which it stands. Thus, *BA* is the base of the right-angled triangle *BAC*. The line drawn from the opposite angle perpendicular to the base, is called the altitude. Thus, *AC* is the altitude.

9. *Name the different kinds of quadrilaterals.*

First.—The *square*, which has all its sides equal, and all its angles right angles.



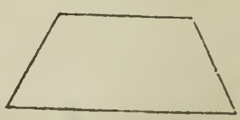
Second.—The *rectangle*, which has its angles right angles, and its opposite sides equal and parallel.



Third.—The *parallelogram*, which has its opposite sides equal and parallel, but its angles not right angles.

Fourth.—The *rhombus*, which has all its sides equal, and the opposite sides parallel, without having its angles right angles.

Fifth.—The *trapezoid*, which has only two of its sides parallel.

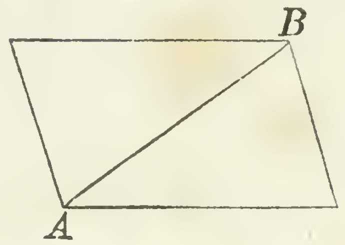


10. *What is the base of a figure? What its altitude?* The base of a figure is the side on which it stands, and

the altitude is a line drawn from the top, perpendicular to the base.

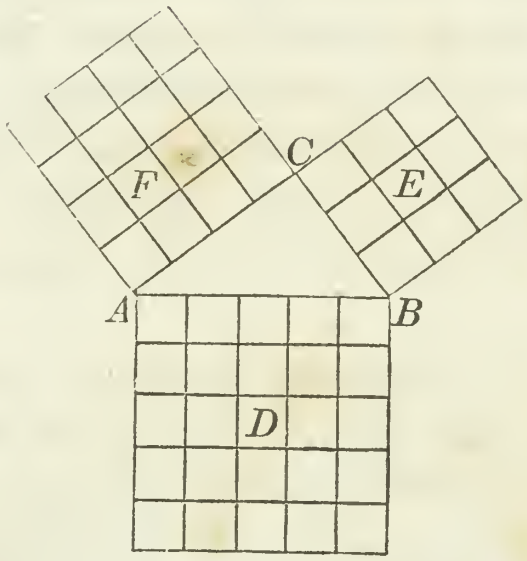
11. *What is a diagonal?*

A diagonal, is a line joining the vertices of two angles not adjacent. Thus, *AB* is a diagonal.



12. *What is the square described on the hypothenuse of a right-angled triangle equal to?*

In every right-angled triangle, the square described on the hypothenuse, is equal to the sum of the squares described on the other two sides.



Thus, if *ABC* be a right-angled triangle, right-angled at *C*, then will the square *D*, described on *AB*, be equal to the sum of the squares *E* and *F*, described on the sides *CB* and *AC*. This is called the carpenter's theorem.

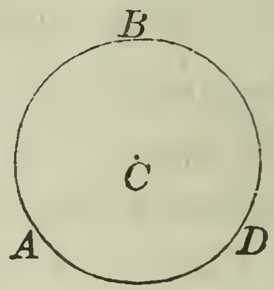
By counting the small squares in the large square *D*, you will find their number equal to that contained in the small squares *F* and *E*.

SECTION III.

OF THE CIRCLE, AND LINES OF THE CIRCLE.

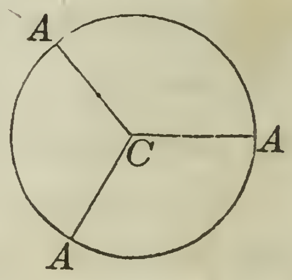
1. *What is a circle? What is a circumference?*

A circle is a plane figure, bounded by a curve line, all the points of which are equally distant from a certain point within, called the centre. The curve line is called the circumference. Thus, the space enclosed by the curve *ABD* is called a circle: the curve *ABD* is the *circumference*, and the point *C*, the centre.



2. *What is the radius of a circle? Are all radii equal?*

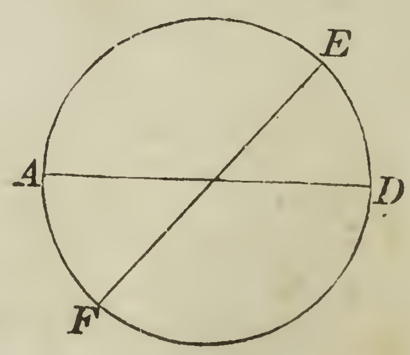
Any line, as *CA*, drawn from the centre *C* to the circumference, is called a *radius*, and two or more such lines, are radii.



All the radii of a circle are equal to each other.

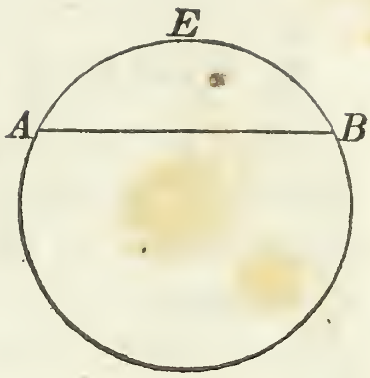
3. *What is the diameter of a circle? How does it divide the circumference?*

The *diameter* of a circle is any line, as *AD*, passing through the centre and terminating in the circumference. Every diameter of a circle divides it into two equal parts, called semicircles, or half circles.



4. *What is an arc?*

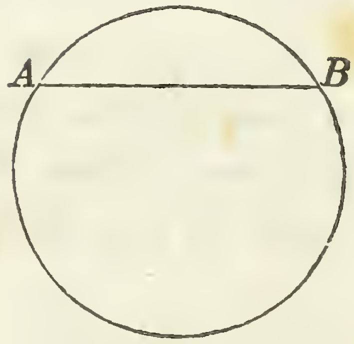
An arc is any portion of the circumference. Thus, *AEB* is an arc.



5. *What is a chord?*

A *chord* of a circle, is a line drawn within a circle, and terminating in the circumference, but not passing through the centre. Thus, *AB* is a chord.

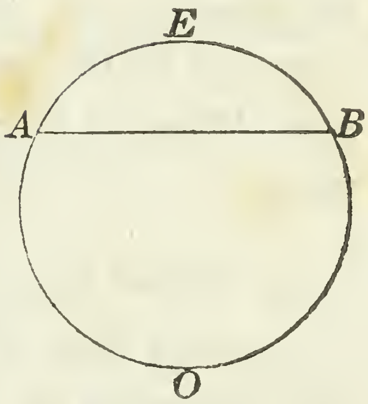
A chord divides the circle into two unequal parts.



6. *What is a segment?*

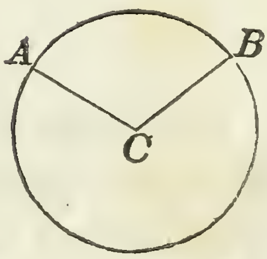
A *segment* of a circle, is a part cut off by a chord. Thus, *AEB* is a segment.

The part *AOB*, is also a segment, although the term is generally applied to the part which is less than a semicircle.



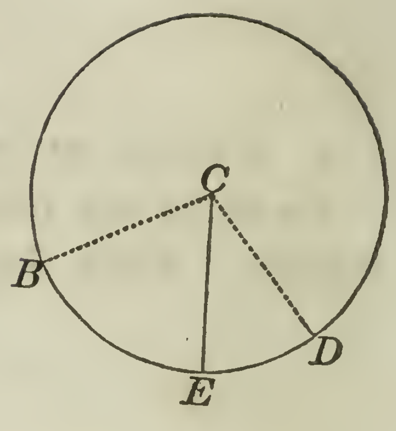
7. *What is a sector?*

A *sector* of a circle, is any part of a circle bounded by two radii and the arc included between them. Thus, *ACB* is a sector.



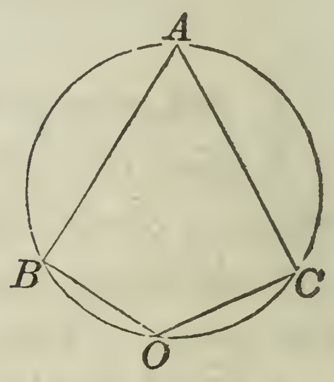
8. *What is an angle at the centre?*

An angle at the centre, is one whose vertex is at the centre of the circle. Thus, *BCE*, or *ECD*, is an angle at the centre.



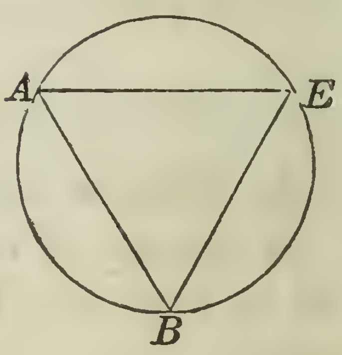
9. *What is an angle at the circumference?*

An angle at the circumference, is one whose angular point is in the circumference. Thus, *BAC*, or *BOC*, is an angle at the circumference.



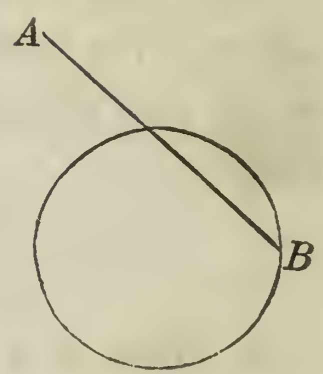
10. *What is an angle in a segment?*

An angle in a segment, is formed by two lines drawn from any point of the segment to the two extrémities of the arc. Thus, *ABE* is an angle in a segment.



11. *What is a secant line?*

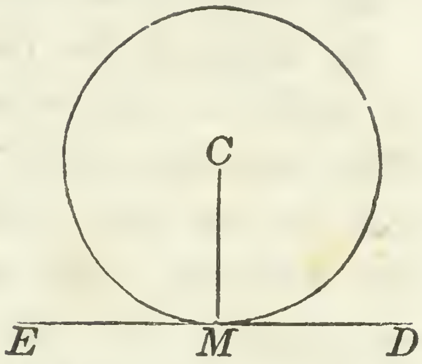
A *secant* line, is one which meets the circumference in two points, and lies partly within and partly without. Thus, *AB* is a secant line.



12. *What is a tangent line?—What position has it with the radius passing through the point of contact?*

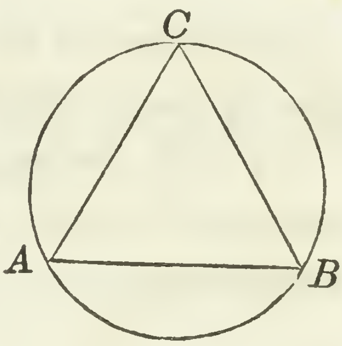
A *tangent* is a line which has but one point in com-

mon with the circumference. Thus, EMD is a tangent. The point M at which the tangent touches the circumference is called the *point of contact*. The tangent line is perpendicular to the radius passing through the point of contact. Thus, CM is perpendicular to EMD.



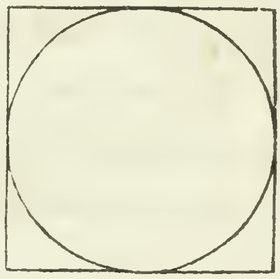
13. *When is a figure said to be inscribed in a circle?—What is said of the circle?*

A figure is said to be inscribed in a circle when all the angular points of the figure are in the circumference. The circle is then said to circumscribe the figure. Thus, the triangle ABC is *inscribed* in the circle, and the circle *circumscribes* the triangle.



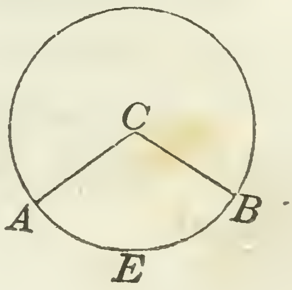
14. *When is a figure said to be circumscribed about a circle?*

A figure is said to be circumscribed about a circle, when all the sides of the figure touch the circumference. The circle is then said to be *inscribed* in the figure.



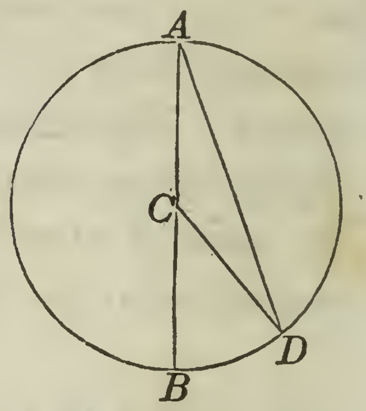
15. *How is an angle at the centre of a circle measured?*

An angle at the centre of a circle is measured by the arc contained by the sides of the angle. This arc is said to *subtend* the angle. Thus, the angle ACB is measured by the degrees in the arc AEB, and is subtended by the arc AEB.



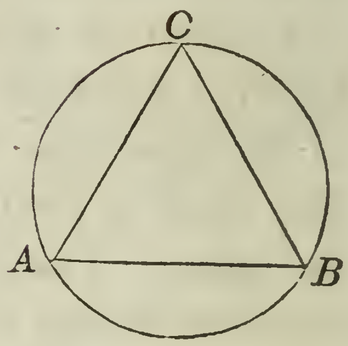
16. *What measures an angle at the circumference?*

An angle at the circumference of a circle, is measured by half the arc which subtends it. Thus, the angle BAD is measured by half the arc BD. Hence, it follows, that when an angle at the centre and an angle at the circumference stand on the same arc BD, the angle at the centre will be double the angle at the circumference.



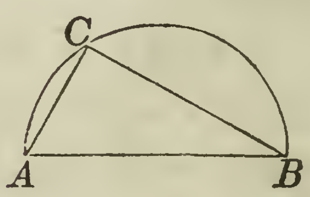
17. *What is the sum of the three angles of any triangle equal to?*

To 180 degrees, since they will be measured by one half of the entire circumference.



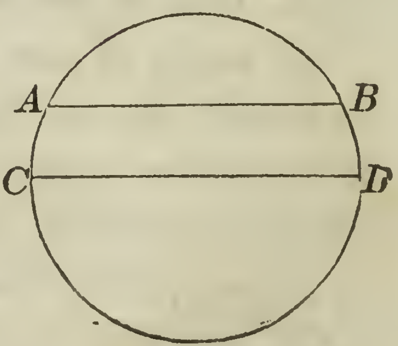
18. *What is an angle in a semicircle equal to?*

An angle inscribed in a semicircle, is a right angle. Thus, if AB be the diameter of a circle, then will the angle ACB be equal to 90 degrees. This angle is measured by one half the semi-circumference, that is, by one half of 180°, or by 90°.

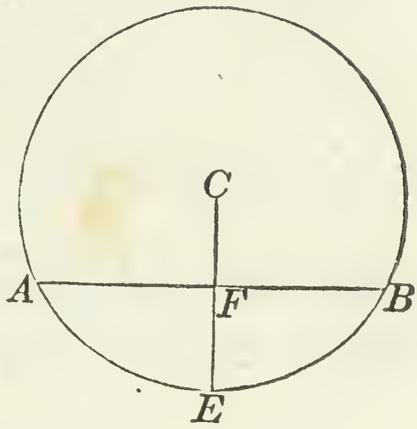


19. *Are the arcs intercepted by parallel chords equal, or unequal?*

Two parallel chords intercept equal arcs. That is, if the chords AB and CD are parallel, the arcs AC and DB, which they intercept, will be equal.



20. *If a line be drawn from the centre of a circle perpendicular to a chord, what follows?*

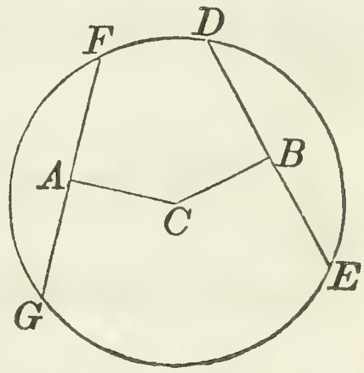


If from the centre of a circle a line be drawn perpendicular to a chord, it will bisect the chord, and also the arc of the chord. Thus, CFE drawn from the centre C, perpendicular to AB, bisects AB at F, and also makes $AE = EB$.

21. *How is the distance from the centre of a circle to a chord measured?*

The distance from the centre of a circle to a chord, is measured on a perpendicular to the chord.

22. *How are chords which are equally distant from the centre?*



In the same, or in equal circles, chords which are equally distant from the centre, are equal. Thus, if $CA = CB$, then will the chord $FG =$ chord DE.

BOOK II.

SECTION I.

PRACTICAL GEOMETRY.

1. *What is Practical Geometry?*

Practical geometry explains the methods of constructing, or describing the geometrical figures.

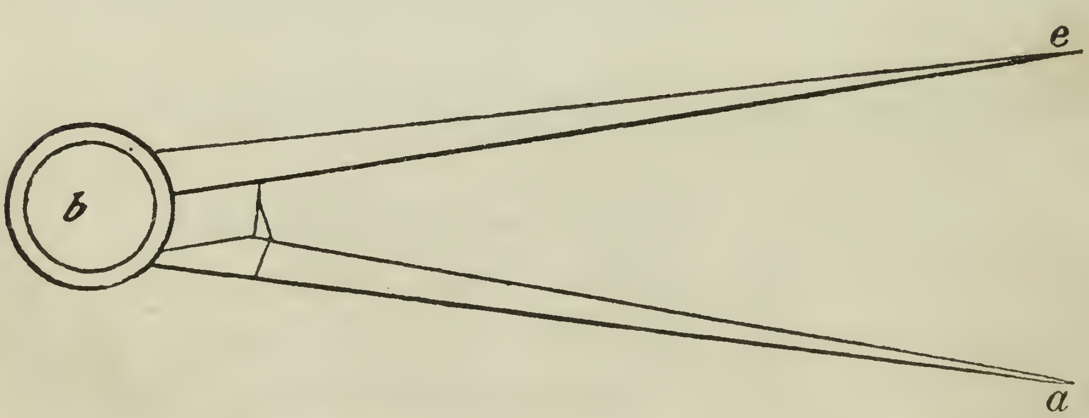
2. *What is a problem?*

Any question which requires something to be done; and doing the thing required, is called *the solution of the problem.*

3. *What are necessary in the solution of geometrical problems?*

Certain instruments which are now to be described.

4. *What are the* DIVIDERS *or* COMPASSES?

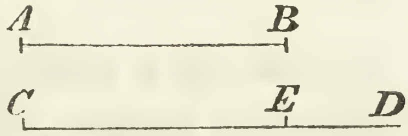


The dividers is the most simple and useful of the in-

struments used for describing figures. It consists of two legs, *ba* and *bc*, which may be easily turned around a joint at *b*.

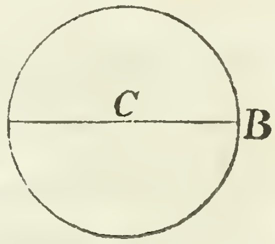
5. *How will you lay off on a line, as CD, a distance equal to AB?*

Take up the dividers with the thumb and second finger, and place the fore-finger on the joint at *b*. Then, set one foot of the dividers at *A*, and extend the other leg with the thumb and fingers, until the foot reaches to *B*. Then, raising the dividers, place one foot at *C*, and mark with the other the distance *CE*, this will evidently be equal to *AB*.



6. *How will you describe from a given centre, the circumference of a circle having a given radius?*

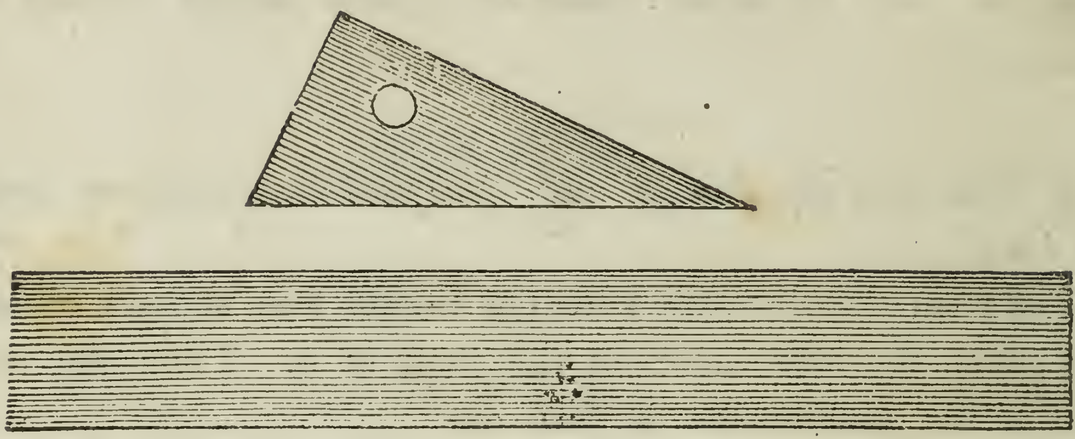
Let *C* be the given centre, and *CB* the given radius. Place one foot of the dividers at *C*, and extend the other leg until it shall reach to *B*. Then turn the dividers around the leg at *C*, and the other leg will describe the required circumference.



7. *How may this be done on a black board with a string and chalk?*

Take one end of the string between the thumb and fore-finger of the left hand, and place it at the centre *C*. Then take the length of the radius on the string, at which point place the chalk held between the thumb and finger of the right hand. Then, holding the end of the string firmly at *C*, turn the right hand around, and the chalk will trace the circumference of the circle.

8. *Describe the* RULER *and* TRIANGLE.

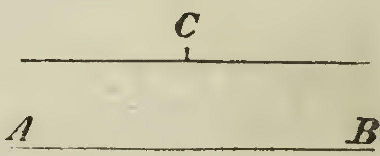


A *ruler* of a convenient size, is about twenty inches in length, two inches wide, and one-fifth of an inch in thickness. It should be made of a hard material, perfectly straight and smooth.

The hypothenuse of the right-angled triangle, which is used in connection with it, should be about ten inches in length, and it is most convenient to have one of the sides considerably longer than the other. We can resolve with the ruler and triangle the two following problems.

9. *Describe the manner of drawing through a given point a line, which shall be parallel to a given line, with the ruler and triangle.*

Let C be the given point, and AB the given line.



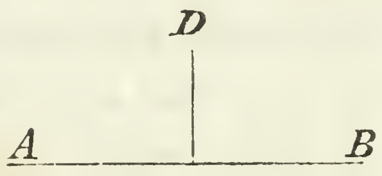
Place the hypothenuse of the triangle against the edge of the ruler, and then place the ruler and triangle on the paper, so that one of the sides of the triangle shall coincide exactly with AB—the triangle being below the line AB.

Then placing the thumb and fingers of the left hand firmly on the ruler, slide the triangle with the other hand along the ruler until the side which coincided with AB reaches the point C. Leaving the thumb of the left hand

on the ruler, extend the fingers upon the triangle and hold it firmly, and with the right hand mark with a pen or pencil a line through C: this line will be parallel to AB.

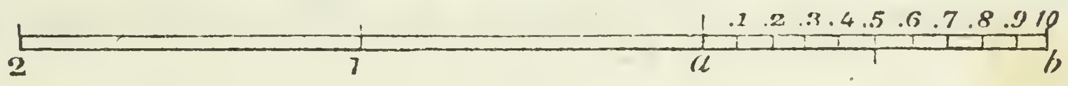
10. *Explain the manner of drawing through a given point, a line which shall be perpendicular to a given line, with the ruler and triangle.*

Let AB be the given line, and D the given point.



Place, as before, the hypothenuse of the triangle against the edge of the ruler. Then place the ruler and triangle so that one of the sides of the triangle shall coincide exactly with the line AB. Then slide the triangle along the ruler until the other side reaches the point D. Draw through D a straight line, and it will be perpendicular to AB.

11. *What is a* SCALE OF EQUAL PARTS?



A scale of equal parts is formed by dividing a line of a given length, into equal portions.

If, for example, the line ab, of a given length, say one inch, be divided into any number of equal parts, as 10, the scale thus formed is called a *scale of ten parts to the inch.*

12. *What is the unit of a scale, and how is it laid off?*

The line ab, which is divided, is called the *unit of the scale.* This unit is laid off several times on the left of the divided line, and the points marked 1, 2, 3, &c. The unit of scales of equal parts, is, in general, either an inch or an exact part of an inch. If, for example, the unit of the scale ab, were one inch, the scale would be one of ten parts to the inch; if it were half an inch, the scale would

be one of ten parts to half an inch, or of 20 parts to the inch.

13. *How will you take from the scale two inches and six-tenths?*

Place one foot of the dividers at 2 on the left, and extend the other to .6, which marks the sixth of the small divisions: the dividers will then embrace the required distance.

14. *How will you lay down, on paper, a line of a given length, so that any number of its parts shall correspond to the unit of the scale?*

Suppose that the given line were 75 feet in length, and it were required to draw it on paper, on a scale of 25 feet to the inch.

The length of the line, 75 feet, being divided by 25, will give 3, the number of inches which will represent the line on paper.

Therefore, draw the indefinite line *AB*, on which lay off

a distance *AC* equal to 3 inches: *AC* will then represent the given line of 75 feet, drawn to the required scale.

15. *What does the last question explain?*

The last question explains the method of laying down a line upon paper, in such a manner that a given number of parts shall correspond to the unit of the scale, whether that unit be an inch or any part of an inch.

When the length of the line to be laid down is given, and it has been determined how many parts of it are to be represented on the paper by a distance equal to the unit of the scale, we find the length which is to be taken from the scale by the following

RULE.

Divide the length of the line by the number of parts which is to be represented by the unit of the scale: the quotient will show the number of parts which is to be taken from the scale.

EXAMPLES.

1. If a line of 640 feet in length is to be laid down on paper, on a scale of 40 feet to the inch; what length must be taken from the scale?

40)640(16 inches.

2. If a line of 357 feet is to be laid down on a scale of 68 feet to the unit of the scale, (which we will suppose half an inch,) how many parts are to be taken?

$$\textit{Ans.} \begin{cases} 5.25, \text{ parts, or} \\ 2.625 \text{ inches.} \end{cases}$$

16. *When the length of a line is given on the paper, how will you find the true length of the line represented?*

Take the line in the dividers and apply it to the scale, and note the number of units, and parts of a unit to which it is equal. Then, multiply this number by the number of parts which the unit of the scale represents, and the product will be the length of the line.

EXAMPLES.

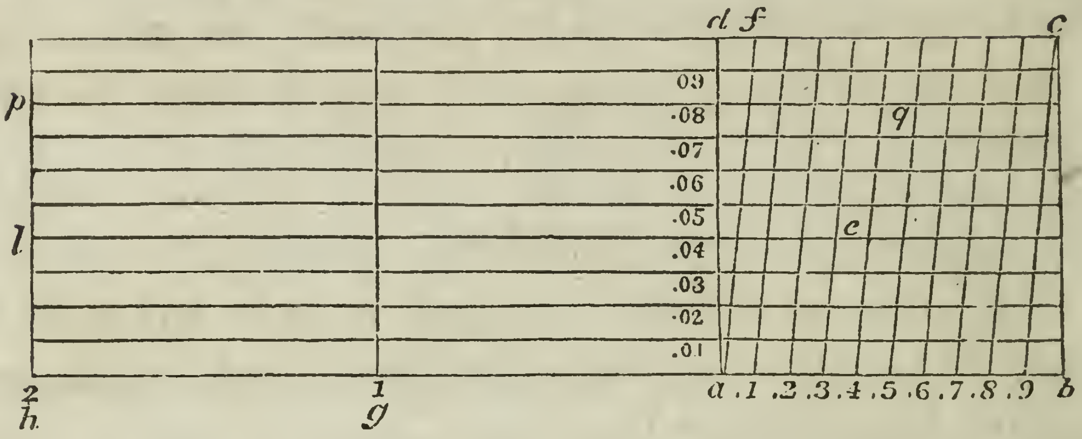
1. Suppose the length of a line drawn on the paper, to be 3.55 inches, the scale being 40 feet to the inch: then,

$3.55 \times 40 = 142$ feet, the length of the line.

2. If the length of a line on the paper is 6.25 inches, and the scale be one of 30 feet to the inch, what is the true length of the line?

Ans. 187.5 feet.

17. *How do you construct the* DIAGONAL SCALE OF EQUAL PARTS?



This scale is thus constructed. Take *ab* for the unit of the scale, which may be one inch, $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{3}{4}$ of an inch, in length. On *ab* describe the square *abcd*. Divide the sides *ab* and *dc* each into ten equal parts. Draw *af* and the other nine parallels as in the figure.

Produce *ab* to the left, and lay off the unit of the scale any convenient number of times, and mark the points 1, 2, 3, &c. Then, divide the line *ad* into ten equal parts, and through the points of division draw parallels to *ab* as in the figure.

Now, the small divisions of the line *ab* are each one-tenth (.1) of *ab*; they are therefore .1 of *ad*, or .1 of *ag* or *gh*.

If we consider the triangle *adf*, the base *df* is one-tenth of *ad*, the unit of the scale. Since the distance from *a* to the first horizontal line above *ab*, is one-tenth of the distance *ad*, it follows that the distance measured on that line between *ad* and *af* is one-tenth of *df*: but since one-tenth of a tenth is a hundredth, it follows that this distance is one hundredth (.01) of the unit of the scale. A like distance measured on the second line will be two hundredths (.02) of the unit of the scale; on the third, .03; on the fourth, .04, &c.

18. *How will you take in the dividers the unit of the scale and any number of tenths?*

Place one foot of the dividers at 1, and extend the other to that figure between *a* and *b* which designates the tenths. If two or more units are required, the dividers must be placed on a point of division farther to the left.

19. *How do you take off units, tenths, and hundredths?*

Place one foot of the dividers where the vertical line, through the point which designates the units, intersects the line which designates the hundredths; then, extend the dividers to that line between *ad* and *bc* which designates the tenths: the distance so determined will be the one required.

For example, to take off the distance 2.34, we place one foot of the dividers at *l*, and extend the other to *e*: and to take off the distance 2.58, we place one foot of the dividers at *p*, and extend the other to *q*.

Remark I.—If a line is so long that the whole of it cannot be taken from the scale, it must be divided, and the parts of it taken from the scale in succession.

Remark II.—If a line be given upon the paper, its length can be found by taking it in the dividers and applying it to the scale.

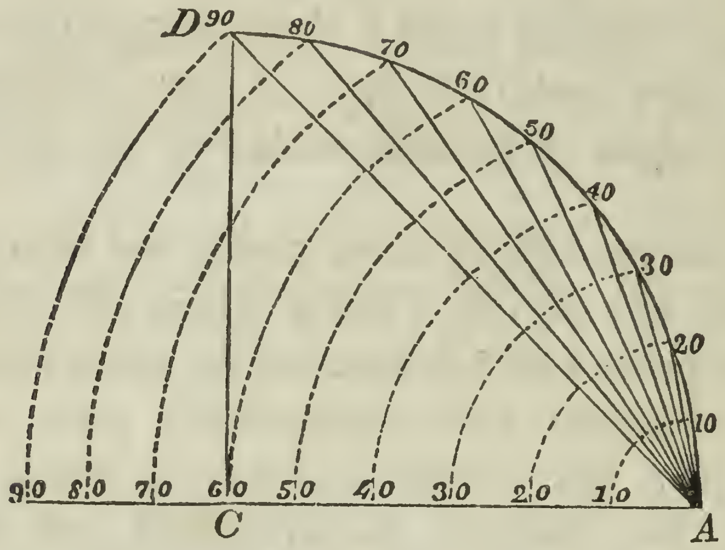
20. *How do you construct a* SCALE OF CHORDS?

If with any radius, as *AC*, we describe the quadrant *AD*, and then divide it into 90 equal parts, each part is called a degree.

Through *A*, and each point of division, let a chord be drawn, and let the lengths of these chords be accurately laid off on a scale: such a scale is called a *scale of chords*. In the figure, the chords are drawn for every ten degrees.

The scale of chords being once constructed, the radius

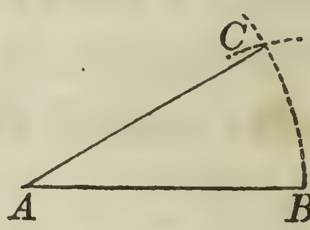
of the circle from which the chords were obtained, is known; for, the chord marked 60 is always equal to the radius of



the circle. A scale of chords is generally laid down on Gunter's scale, and on the scales which belong to cases of mathematical instruments, and is marked CHO.

21. *How will you lay off an angle with a scale of chords; say an angle of* 30 *degrees?*

Let *AB* be the line from which the angle is to be laid off, and *A* the angular point.



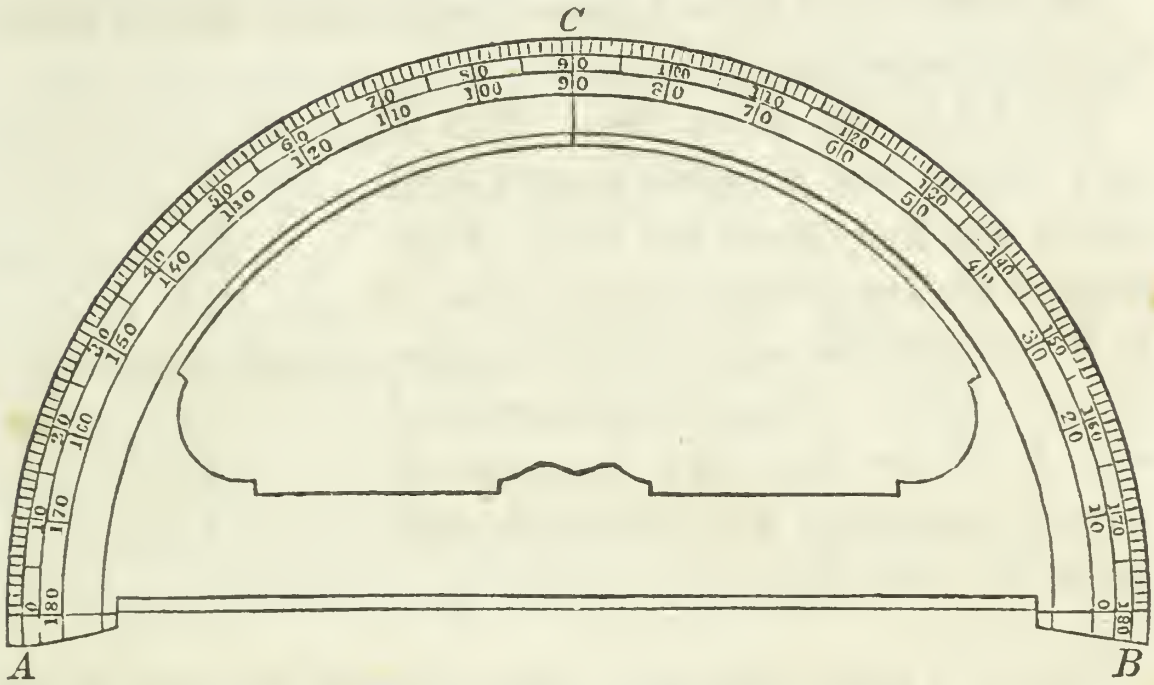
Take from the scale, the chord of 60 degrees, and with this radius and the point *A* as a centre, describe the arc *BC*. Then take from the scale the chord of the given angle, say 30 degrees, and with this line as a radius, and *B* as a centre, describe an arc cutting *BC* in *C*. Through *A* and *C* draw the line *AC*, and *BAC* will be the required angle.

22. *Describe the* SEMICIRCULAR PROTRACTOR.

This instrument is used to lay down, or protract angles. It may also be used to measure angles included between lines already drawn upon paper.

It consists of a brass semicircle *ACB*, divided to half

degrees. The degrees are numbered from 0 to 180, both ways; that is, from *A* to *B*, and from *B* to *A*. The di-



visions, in the figure, are only made to degrees. There is a small notch at the middle of the diameter *AB*, which indicates the centre of the protractor.

23. *How do you lay off an angle with a protractor?*

Place the diameter *AB* on the line, so that the centre shall fall on the angular point. Then count the degrees contained in the given angle from *A* towards *B*, or from *B* towards *A*, and mark the extremity of the arc with a pin. Remove the protractor, and draw a line through the point so marked and the angular point: this line will make with the given line the required angle.

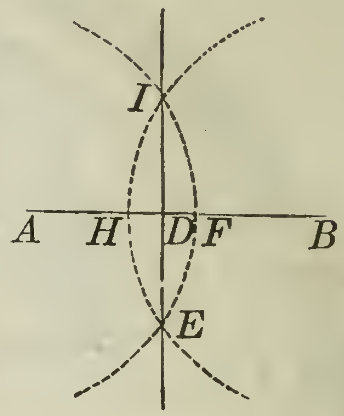
24. *Describe* GUNTER'S SCALE.

This is a scale of two feet in length, on the faces of which, a variety of scales are marked. The face on which the divisions of inches are made, contains, however, all the scales necessary for laying down lines and angles. These are, the Scale of Equal Parts, the Diagonal Scale of Equal

Parts, and the Scale of Chords, all of which have been described.

25. *How do you bisect a given straight line; that is, divide it into two equal parts?*

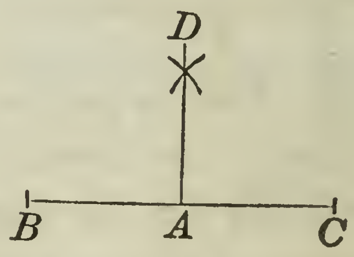
Let AB be the given line. With A as a centre, and a radius greater than half of AB, describe an arc IFE. Then remove the foot of the dividers from A to B, and with the same radius describe the arc EHI. Then join the points I and E by the line IE: the point D, where it intersects AB, will be the middle of the line AB.



26. *At a given point in a given straight line, how do you draw a perpendicular to that line?*

Let A be the given point, and BC the given line.

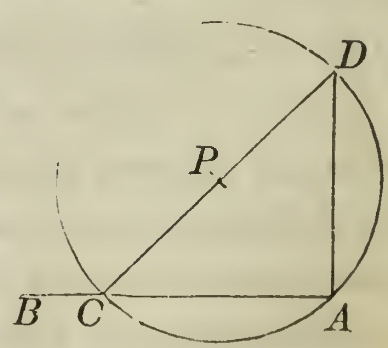
From A lay off any two distances AB and AC, equal to each other. Then, from the points B and C, as centres, with a radius greater than BA, describe two arcs intersecting each other in D: draw AD, and it will be the perpendicular required.



SECOND METHOD.

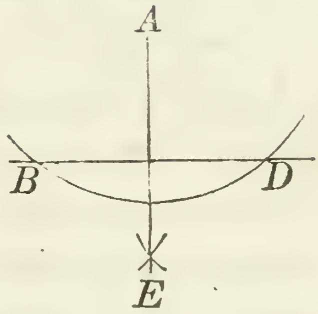
27. *When the point A is near the end of the line.*

Place one foot of the dividers at any point, as P, and extend the other leg to A. Then with P as a centre, and radius from P to A, describe the circumference of a circle. Through C, where the circumference cuts BA, and the centre P, draw the line CPD.



Then draw AD, and it will be perpendicular to CA, since CAD is an angle in a semicircle.

28. *Draw from a given point without a straight line, a perpendicular to that line.*

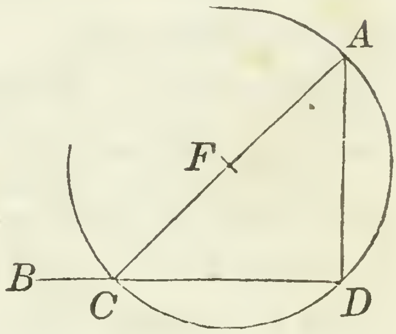


Let A be the given point, and BD the given line.

From the point A as a centre, with a radius sufficiently great, describe an arc cutting the line BD in the two points B and D: then mark the point E, equally distant from the points B and D, and draw AE: and AE will be the perpendicular required.

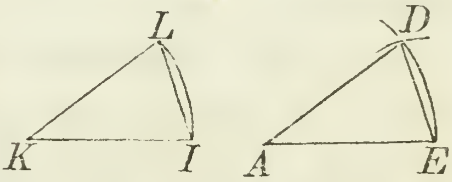
SECOND METHOD.

29. *When the given point A, is nearly opposite one end of the given line.*



Draw AC to any point, as C of the line BD. Bisect AC at F. Then with F as a centre, and FC or FA as a radius, describe the semicircle CDA. Then draw DA, and it will be perpendicular to BD at D.

30. *At a point, in a given line, to make an angle equal to a given angle.*

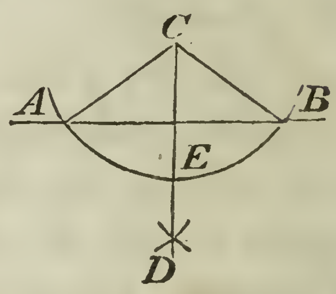


Let A be the given point, AE the given line, and IKL the given angle.

From the vertex K, as a centre, with any radius, describe the arc IL, terminating in the two sides of the angle. From the point A as a centre, with a distance AE equal to KI, describe the arc ED; then take the chord LI, with which, from the point E as

a centre, describe an arc cutting the indefinite arc DE, in D: draw AD, and the angle EAD will be equal to the given angle K.

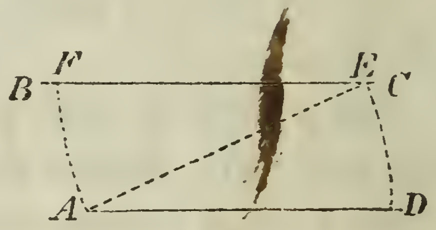
31. *How do you divide a given angle, or a given arc, into two equal parts?*



Let C be the given angle, and AEB the arc which measures it.

From the points A and B as centres, describe with the same radius two arcs cutting each other in D: through D and the centre C draw CD: the angle ACE will be equal to the angle ECB, and the arc AE to the arc EB.

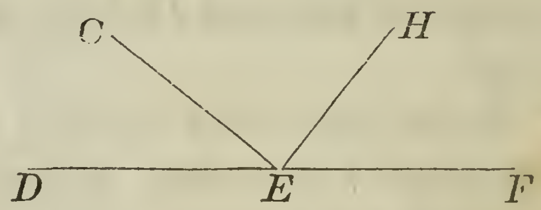
32. *How do you draw through a given point a line parellel to a given line?*



Let A be the given point, and BC the given line.

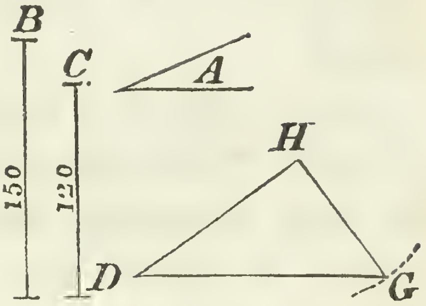
From A as a centre, with a radius greater than the shortest distance from A to BC, describe the indefinite arc ED: from the point E as a centre, with the same radius, describe the arc AF; make $ED = AF$, and draw AD: then will AD be the parallel required.

33. *If two angles of a triangle are given, how do you find the third?*



Draw the indefinite line DEF. At the point E, make the angle DEC equal to one of the given angles, and then the angle CEH equal to the other: the remaining angle HEF will be the third angle required.

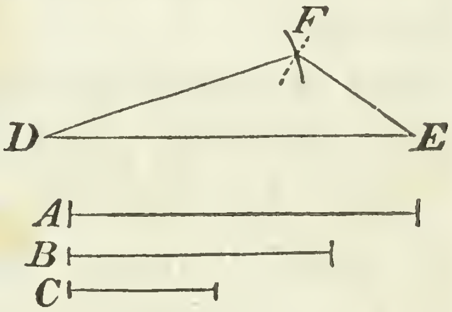
34. *If two sides and the included angle of a triangle are given, how do you describe the triangle?*



Let the line $B = 150$ feet, and $C = 120$ feet, be the given sides; and $A = 30$ degrees, the given angle: to describe the triangle on a scale of 200 feet to the inch.

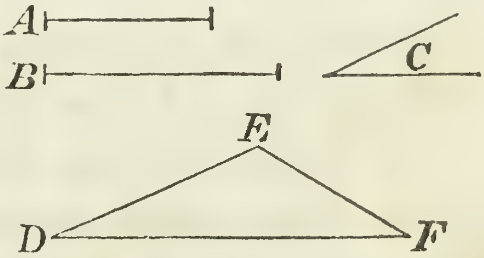
Draw the indefinite line DG, and at the point D, make the angle GDH equal to 30 degrees; then lay off DG equal to three quarters of an inch, and it will represent the side $B = 150$ feet: make DH equal to six-tenths of an inch, and it will represent $C = 120$ feet: then draw GH, and GDH will be the required triangle.

35. *If the three sides of a triangle are given, how do you describe the triangle?*



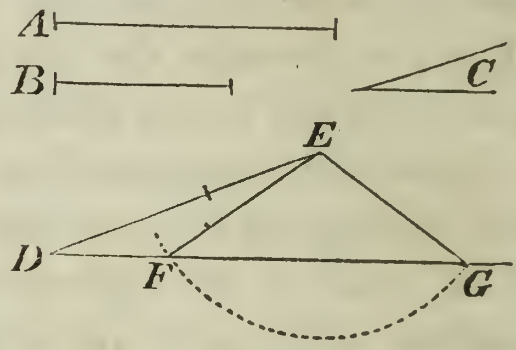
Let A, B, and C, be the sides. Draw DE equal to the side A. From the point D as a centre, with a radius equal to the second side B, describe an arc: from E as a centre, with a radius equal to the third side C, describe another arc intersecting the former in F; draw DF and EF, and DEF will be the triangle required.

36. *If two sides of a triangle and an angle opposite one of them are given, how do you describe the triangle?*



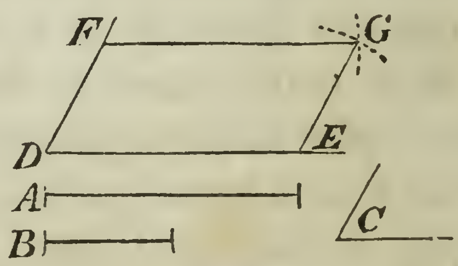
Let A and B be the given sides, and C the given angle, which we will suppose is opposite the side B. Draw the indefinite line DF, and make the angle FDE equal to the angle

C: take $DE = A$, and from the point E as a centre, with a radius equal to the other given side B, describe an arc cutting DF in F; draw EF: then will DEF be the required triangle.



If the angle C is acute, and the side B less than A, then the arc described from the centre E with the radius $EF = B$ will cut the side DF in two points, F and G, lying on the same side of D: hence there will be two triangles, DEF and DEG, either of which will satisfy all the conditions of the problem.

37. *If the adjacent sides of a parallelogram, with the angle which they contain, are given, how do you describe the parallelogram?*



Let A and B be the given sides, and C the given angle.

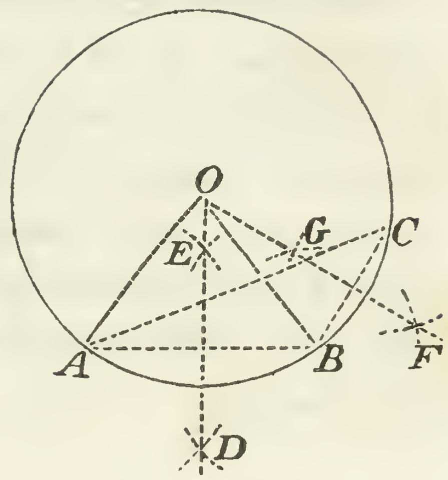
Draw the line $DE = A$; at the point D, make the angle $EDF = C$; take $DF = B$: describe two arcs, the one from F as a centre, with a radius $FG = DE$, the other from E, as a centre, with a radius $EG = DF$; through the point G, where these arcs intersect each other, draw FG, EG; then $DEGF$ will be the parallelogram required.

38. *How do you describe the circumference of a circle which shall pass through three given points?*

Let A, B, and C, be the three given points.

Join these points by straight lines AB, BC, CA. Then bisect any two of these straight lines by the perpendicu-

lars OF, OD, as in Section 25, and the point O, where these perpendiculars intersect each other, will be the centre of the circle.



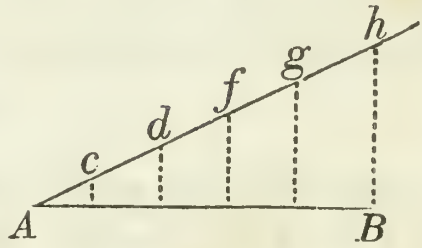
Place one foot of the dividers at this centre, and extend the other to A, B, or C, and then with this radius, let the circumference be described.

39. *How do you find the centre of a circle when the circumference is given?*

Take any three points, as A, B, and C, (see last figure,) and join them by the lines AB and BC. Then bisect these lines by the perpendiculars OD and OF, and O will be the centre of the circle.

40. *How do you divide a given line AB, into any number of equal parts?*

Let AB be the given line to be divided. Let it be required, if you please, to divide it into five equal parts.

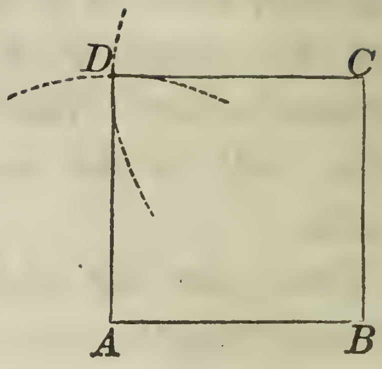


Throughout A, one extremity of the line, draw$A$ h, making an angle with AB. Then lay off on Ah, five equal parts, Ac, cd, df, fg, gh, after which join h and B. Through the points of division c, d, f, and g, draw lines parallel to hB, and they will divide AB into the required number of equal parts.

41. *How do you describe a square on a given line?*

Let AB be the given line. At the point B, draw BC perpendicular to AB, and then make it equal to BA.

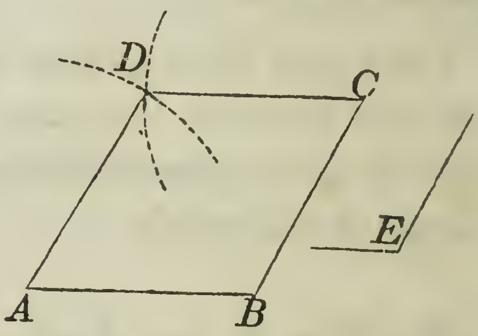
Then, with A as a centre, and radius equal to AB, describe an arc; and with C as a centre, and the same distance AB, describe another arc, and through D, their point of intersection, draw AD and CD; then will $ABCD$ be the required square.



41. *How do you construct a rhombus, having given the length of one of the equal sides and one of the angles?*

Let AB be equal to the given side, and E, the given triangle.

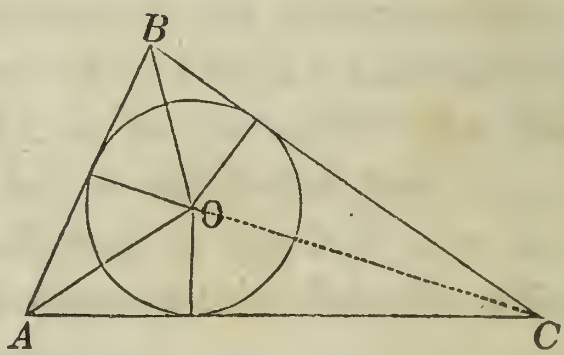
At B, lay off an angle ABC, equal to E, and make BC equal to AB. Then with A and C as centres, and a radius equal to AB, describe two arcs, and through D, their point of intersection, draw the lines AD and CD, and $ABCD$ will be the required rhombus.



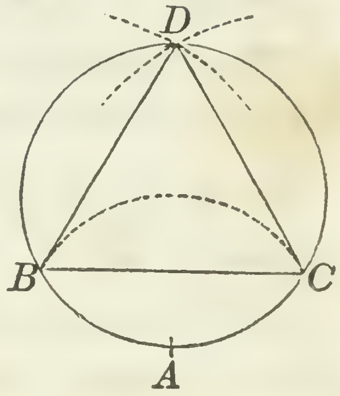
42. *How do you inscribe a circle in a given triangle?*

Let ABC be the given triangle.

Bisect either two of the angles, as A and C, by the lines AO and CO, and the point of intersection O will be the centre of the inscribed circle. Then, through the point of intersection O, draw a line perpendicular to either side, and it will be the radius.

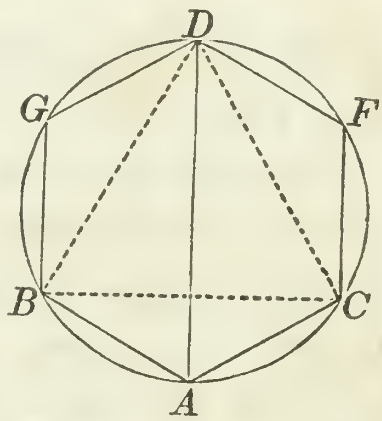


43. *How do you inscribe an equilateral triangle in a circle?*



With any point A, as a centre, and radius equal to the radius of the circle, describe an arc cutting the circumference in B and C. Then bisect the arc BDC, after which, draw BC, BD, and CD, and BDC will be an equilateral triangle.

44. *How do you inscribe a hexagon in a circle?*

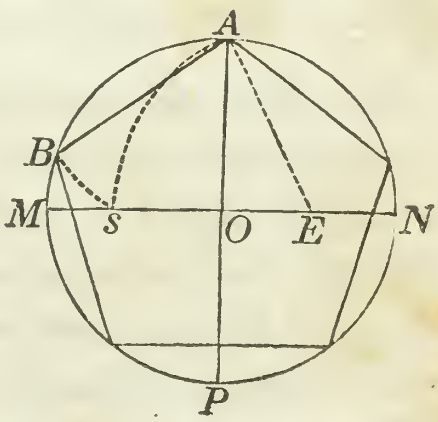


Describe the equilateral triangle as before. Then bisect the arc CD in F, and the arc BD at G, and draw AC, CF, FD, DG, GB, and BA, and $ACFDGB$ will be the hexagon required. Or the hexagon may be inscribed by applying the radius six times around the circumference.

45. *How do you inscribe a dodecagon in a circle?*

Bisect the arcs which subtend the chords of the hexagon, and through the points of bisection draw chords, and there will be formed a regular dodecagon.

46. *How do you inscribe in a circle a regular pentagon?*



Draw the diameters AP and MN at right angles to each other, and bisect the radius ON at E. From E as a centre, and EA as a radius, describe the arc As; and from the point A as a centre, and radius As, describe the arc sB.

Join the points A and B, and the line AB, being applied five times around the circle, will form the required pentagon.

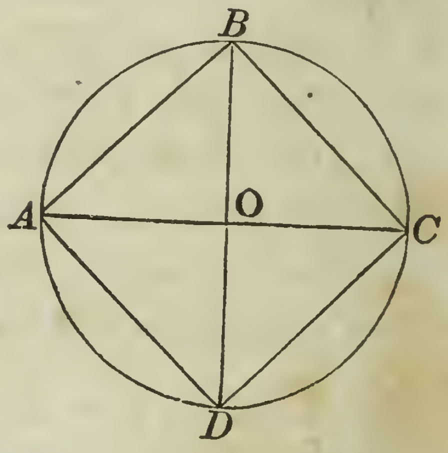
47. *How do you inscribe in a circle a regular decagon?*

For the decagon, bisect the arcs which subtend the sides of the pentagon, and join the points of bisection; and the lines so drawn will form the regular decagon.

48. *How will you inscribe in a circle a polygon having any number of sides?*

Divide the circumference of the circle into as many equal parts as there are sides of the polygon, and draw lines through the points of division: these lines will be the sides of the required polygon.

49. *How do you inscribe a square in a given circle?*

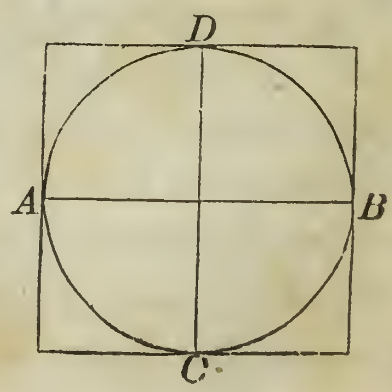


Let $ABCD$ be the given circle. Draw two diameters DB and AC at right angles to each other, and through the points A, B, C, and D, draw the lines AB, BC, CD, and DA: then $ABCD$ will be an inscribed square.

50. *How would you inscribe an octagon?*

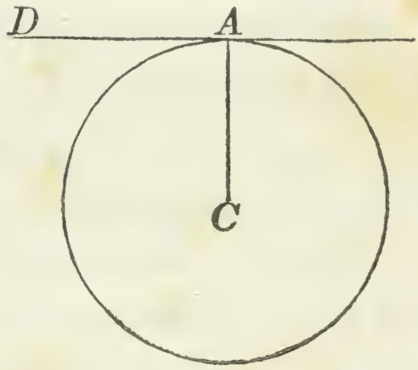
By bisecting the arcs AB, BC, CD, and DA, and joining the points of bisection, we can form an octagon; and by bisecting the arcs which subtend the sides of the octagon, we can inscribe a polygon of sixteen sides.

51. *How will you circumscribe a square about a circle?*



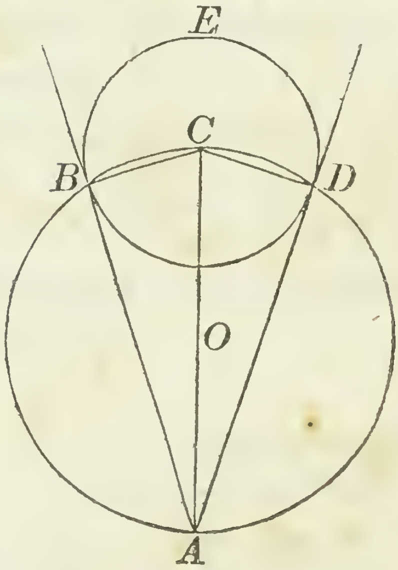
Draw two diameters AB and CD at right angles to each other; and through their extremities A, B, C, and D, draw lines respectively parallel to the diameters CD and AB: a square will thus be formed circumscribing the circle.

52. *How do you draw a line which shall be tangent to the circumference of a circle at a given point?*



Let A be the given point. Through A draw the radius AC, and then draw DA perpendicular to the radius at the extremity A. The line DA will be tangent to the circumference at the point A.

53. *How do you draw through a given point without a circle a line which shall be tangent to the circumference?*



Let A be the given point without the given circle BED. Join the centre C and the given point A, and bisect the line CA at O.

With O as a centre, and OA as a radius, describe the circumference $ABCD$. Through B and D draw the lines AB and AD, and they will be tangent to the circle BED at the points B and D.

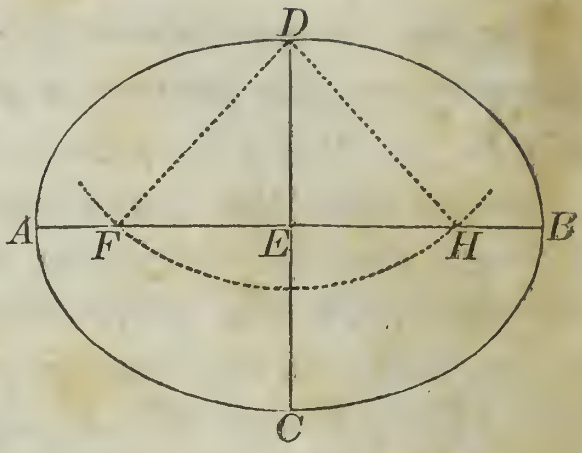
54. *What is an ellipse?*

It is an oval curve $ACBD$.

55. *What is the longest line which can be drawn within the curve called? What is the shortest line called?*

The longest line AB is called the *transverse axis*; and the shortest line DC is called the *conjugate axis*. The point E, at which they intersect, is called the centre of the ellipse.

56. *What are the foci of an ellipse?*



They are two points F and H, determined by describing the arc of a circle with D as a centre, and a radius DF equal to AE, half of the transverse axis.

57. *How will you describe an ellipse when you know the two axes AB and CD?*

First, find the foci F and H by describing an arc with D as a centre, and with a radius equal to AE.

Secondly, take a string or thread equal in length to AB, and fasten the extremities at the foci F and H. Then place a pencil against the string and move it round, bearing it tight against the string, and the point will describe the ellipse $ADBC$.

QUESTIONS TO BE PUT FROM FIGURES MADE BY THE TEACHER UPON THE BLACK-BOARD.

SECTION I.

WHAT is a line?
What is a right line?
What is a curve?
What does the word *line* imply?
What is a surface?
What is a plane?
What are parallel right lines?
What are parallel curves?
What are oblique lines?
What are horizontal lines?
What are vertical lines?
What is an angle? how read?
What are curvilinear angles?
When is one line perpendicular to another?
What are the angles then called?
What is an acute angle?
What is an obtuse angle?
What follows when two lines intersect each other?
What follows when one line cuts two parallels?
What follows when one line is perpendicular to one of several parallels?
How many lines can be drawn from a point perpendicular to a given line?
If oblique lines are drawn, how do they compare?
What is the circumference of a circle?

once kissed a gal, and it was such a smack,
For I loved it; but then here's the joke;
Her mother from the bed screamed at hearing the crac
"There! Patty, my yeast bottle's broke."

For what is it used?
How is it divided?
How are angles measured?
How many degrees in one right angle?
What is one quarter of the circumference called? One half?
When one straight line meets another, what is the sum of the angles on the same side?
If there are several angles, what is their sum equal to?
What is the sum of all the angles about a given point equal to?

SECTION II.

What is a plane figure?
What is it called when the bounding lines are straight?
What are the lines themselves called?
What is a triangle?
What is a quadrilateral?
What is a polygon of five sides?
What is a polygon of six sides?
What is a polygon of seven sides?
What is a polygon of eight sides?
What is a polygon of nine sides?
What is a polygon of ten sides?
What is a polygon of twelve sides?
What is the smallest number of straight lines which can enclose a space?
What are the several kinds of triangles?
What is the base of a triangle?
What its altitude?
What are the different kinds of quadrilaterals?
What is the base of a figure?
What is a diagonal?
What is the square described on the hypothenuse of a right-angled triangle equal to?

SECTION III.

What is a circle?
What is a circumference?
What is the radius of a circle?
What is an arc?
What is a chord?
What is a segment?
What is a sector?
What is an angle at the centre?
What is an angle at the circumference?
What is an angle in a segment?
What is a secant line?
What is a tangent line?
What position has the tangent with the radius?
When is a figure said to be inscribed in a circle?
When circumscribed about it?
How is an angle at the centre of a circle measured?
What measures an angle at the circumference?
What is the sum of the three angles of a triangle equal to?
How does a perpendicular through the centre divide the chord?
How do the distances from the centre to equal chords compare with each other?

PRACTICAL GEOMETRY.

What is Practical Geometry?
What is a problem?
What are the dividers?
How do you lay off a line?
How do you describe the circumference of a circle?
How on the black-board?
Describe the ruler and triangle, and the manner of using them.
How do you draw a perpendicular?
What is a Scale of Equal Parts?
What is a unit of the scale?
Explain how you take from the scale a given number of parts.
Explain the Diagonal Scale.
What is a Scale of Chords?
How will you lay off an angle?
What is the Semicircular Protractor?
How do you lay off an angle with it?
Describe Gunter's Scale.
How do you bisect a line?
How do you draw a perpendicular at a given point?
How do you make an angle equal to a given angle?
How do you bisect an arc?
How do you draw a parallel to a given line?
When two angles of a triangle are given, how do you find the third?
When two sides and the included angle are given, how do you describe the triangle?
How do you describe a parallelogram with the same given?
How do you pass the circumference of a circle through three points?
How do you divide a line into any number of equal parts?
How do you describe a square?
How do you construct a rhombus?
How do you inscribe a circle in a given triangle?
How do you inscribe an equilateral triangle in a given circle?
How do you inscribe a hexagon in a circle?
How do you inscribe a dodecagon?
How do you inscribe in a circle a polygon having any number of sides?
How do you inscribe a square? an octagon?
How do you circumscribe a square about a circle?
How do you draw a line tangent to a circle at a point of the circumference?
How from a point without the circumference?

Note.—After the teacher shall have made the above figures, or most of them, on the black-board, and the pupils copied them on their slates, let the students then be called to the black-board in turn, and practised in the drawing of them.

BOOK III.

SECTION I.

OF DRAWING IN GENERAL.

1. *What are drawings?*

DRAWINGS are representations to the eye of the forms, dimensions, positions, and appearance of objects. They form a *written language*, which is easily comprehended by every one.

2. *What are the uses of drawing?*

Drawing, to the practical man, furnishes a simple means of describing and explaining a thing in a brief and striking manner. On this account, alone, its great advantages are everywhere apparent. Drawings, also, impress the mind with images approaching nearer to the reality, than any other means of description. The pen of the ablest historian presents but a feeble image, when compared with the pictured canvass of the painter, or the life-like forms of the sculptor.

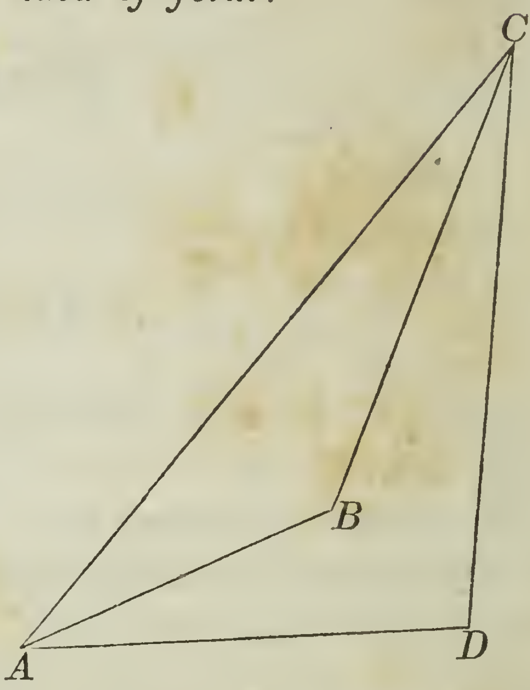
3. *When you look at a single object, what do you observe that distinguishes it from other objects?*

When we observe a single object, we discover that we are able to recognise it by means of three properties which distinguish it from other objects, viz.: its *form*, its *light and shade*, and its *color*. If we consider more than one

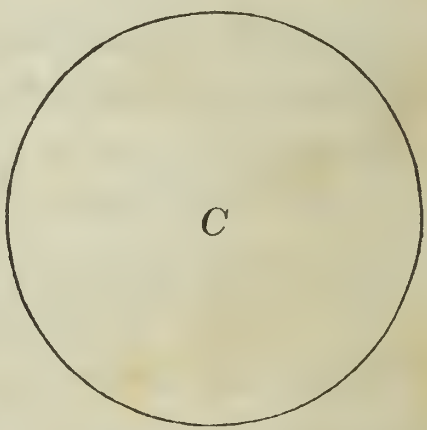
object, we are then able to distinguish them from each other, by their *relative position*, also.

4. *How do you illustrate the idea of form?*

If we join any three points *A*, *B*, and *C*, by straight lines, the result will be a *figure* or *form* of a triangle. If we take another point *D*, and join the three points *A*, *D*, and *C*, we shall have the form of another triangle *ADC*. The straight lines which bound each of these figures, make up what is called its *outline.*



If with *C* as a centre, and any radius, we describe the circumference of a circle, the curve so drawn will be the outline of the circle.



Now, the triangle and circle differ from each other only in *form*, and the form is determined by the *outline:* hence we see that *outline* is one means of representing form to the eye. It is thus that we are able to distinguish a triangle from a circle, and a circle from a square; and the drawings of their outlines present to the mind, through the eye, the idea of the objects themselves.

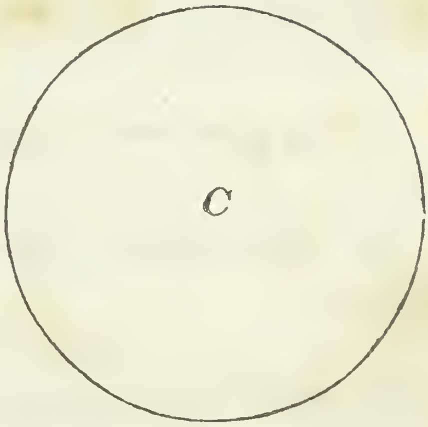
5. *How do you illustrate light and shade?*

If we hold any object in the sun's rays, it is evident, that that part of it which is turned towards the sun will be lighted; and that the part which is turned away from the

sun will be comparatively dark. The part towards the sun is called the *light;* the other part, the *shade.*

6. *In what manner do light and shade modify the idea of a form which is represented only by its outline?*

The circle whose centre is *C*, is the outline of so much of the flat white paper as is contained within its circumference.

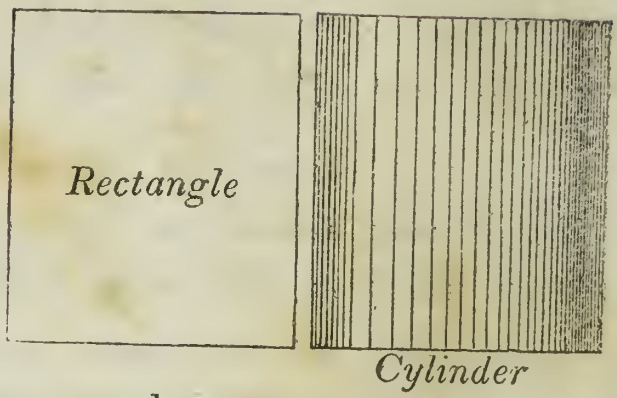


Now, if we observe a sphere, or perfectly round ball, we find that, in every position, its outline is also a circle. We cannot tell, therefore, whether this circle is the outline of a circular piece of paper or of a sphere.

Let the circle whose centre is *D*, be the outline of a sphere. If we suppose the light to proceed from the left hand, then the part of the sphere towards the left will be the light, and the part towards the right, the shade.

Leaving the white paper for the light, we will represent the shade, or dark part, by means of lines drawn in such a manner, as to darken that part occupied by the shade.

In a similar manner, the outline of a rectangle may be distinguished from that of a cylinder by means of light and shade. Thus we see that light and shade furnish a distinction between objects whose outlines are the same.



7. *In how many ways may the shade on a body be modified?*

In two ways: viz, in its depth or intensity, and its color.

8. *How do you know which part of a body has the greatest depth or intensity of shade?*

If there were no atmosphere, and no body in existence except the one we are considering, that part of it which does not receive the sun's rays would be invisible. But since the atmosphere, as well as every other substance in nature, reflects back the light which it receives, casting it in a direction contrary to that of the sun's rays; it follows, that the part of any object which does not receive the direct light of the sun, will yet receive light from other objects, behind it with reference to the sun, and will be sufficiently illuminated to exhibit its form. Now, since bodies are more or less illuminated as they receive the light directly or obliquely, it follows, that if we conceive the reflecting body to be placed directly behind the one receiving the light, that the part nearest the reflecting body will receive more light than the parts more remote; and hence, the shade there will be less intense. It therefore follows, that the effect of *reflected light* on the depth of shade, will be the greatest near the outline of the body which is farthest from the source of light.

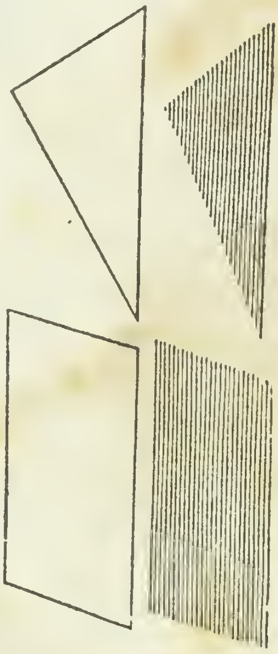
9. *How may the shade of an object be modified in regard to color?*

Every reflected ray of light is of the same color as the body which reflects it, and when such rays illuminate a dark object, they also impart to it their color. This may be shown by holding any dark body, as a sphere, in the sun's rays, and placing near it, and opposite to the sun, a piece of bright-colored red or yellow paper. The reflected rays from the paper will impart their tint to the shade of the sphere.

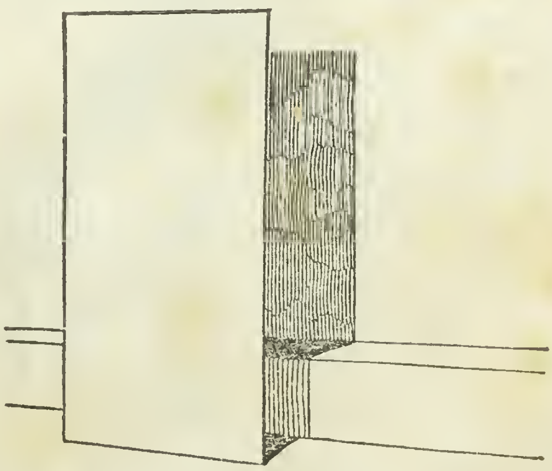
10. *What is the shadow on a body?*

The shadow on a body is that part of it from which the light is intercepted by some opaque body.

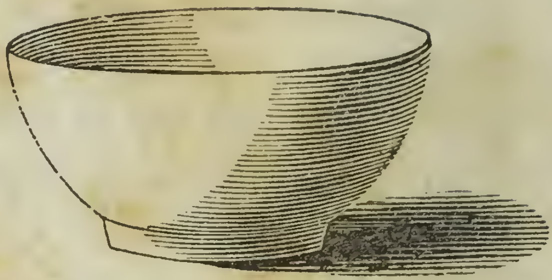
11. *How may the forms of objects be discovered by means of the shadows which they cast or receive?*



It is evident that the shadow of a triangle, or of a square, on a flat surface, will, in certain positions, exactly resemble the bodies which cast them. But the surface which receives the shadow will modify the shape of it; and thus the shadow will also give an idea of the form of the surface on which it falls.

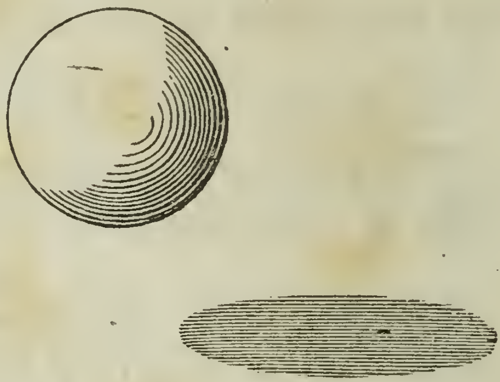


For example, the rectangle in the figure casts a shadow of such a shape on the wall and step which are behind it, as to show their form distinctly. Without the shadow, the two lines which are the outlines of the step, might equally well represent two horizontal lines drawn upon the wall.



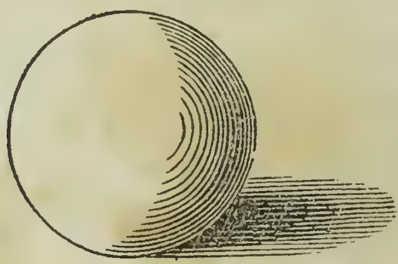
This example exhibits the outline, light and shade, reflection, and shadow of a cup; and is an illustration of the foregoing principles.

12. *How may the relative position of objects be determined by the shadows which they cast or receive?*

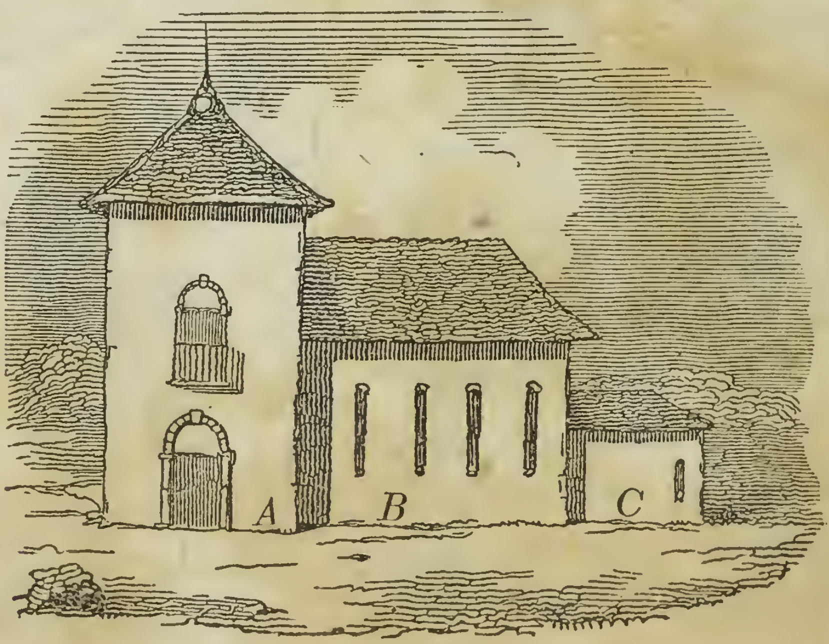


When a shadow is entirely separated from the body which casts it, as is the shadow of the sphere in this example, it is then plain that a space intervenes between the body and the surface on which the shadow falls.

But when the shadow joins the body which casts it, as in this example, then the body casting the shadow touches the surface on which the shadow falls.

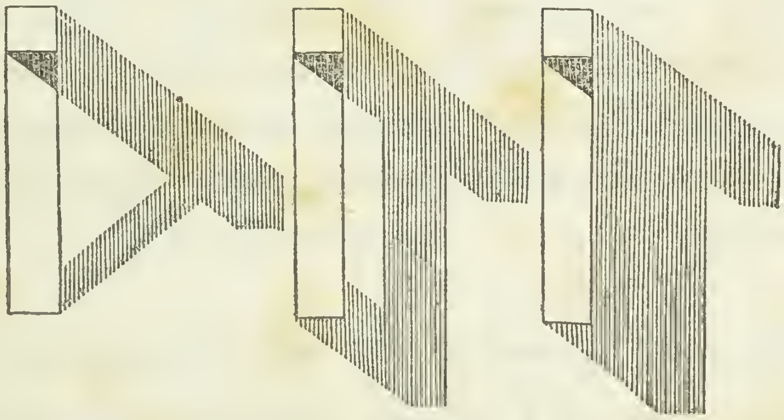


Hence, the nearer an object is to the surface on which the shadow falls, the nearer will the shadow approach to the object.



The example of the house shows, by the shadows on *B* and *C*, that *B* stands further back than *A*, and *C* farther than *B*.

The shadows in the example which follows, exhibit the difference between the forms of three objects whose outlines are exactly the same. The shade on them cannot be represented in these outlines.



13. *What may be said of color, as a means of distinguishing objects from each other?*

Of this, it is only necessary to observe, that when we have represented the form of an object, its light and shade, and its shadow, if we wish still further to distinguish it from other objects, we have but to add its appropriate color. For example, in the drawing of a machine, if we wish to exhibit the difference between the wood, the iron, and the brass, the natural colors of these should be added in the drawing.

14. *What effect have shade and shadow?*

Shade and shadow have the effect of obscuring the outline, form, and color, of that part of every object on which they are found. Hence shading, in drawing, is the obscuring, in imitation of nature, of those portions of the objects we are representing, and from which the light is intercepted. There is this difference, however, between nature and art:—in the former we distinguish and deter-

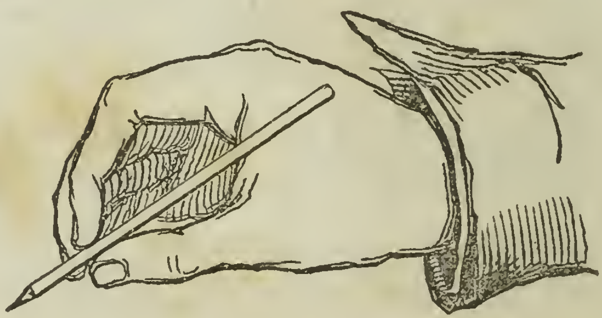
mine forms by means of the light; in the latter, by the shade and shadow.

15. *By what is the process of shading regulated?*

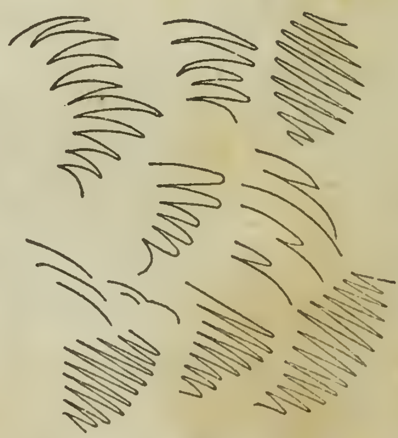
The process of shading a drawing varies according to the instrument used. The pen is capable of making only lines and dots; hence, if we employ it only, we are confined to those two methods of shading. The brush and lead pencil possess, in addition to the resources of the pen, the capability of laying a smooth, graduated tint of shade, which by the brush may also be made of any color that may be desired.

16. *What may be said of the use of the pencil?*

The acquisition of a skilful and easy manner of handling the pencil, depends in a great measure upon the way of holding it. The thumb, with the first and second fingers, should grasp the pencil about an inch from its point. The thumb should not be drawn back, as we are taught in holding a pen for writing; but should be placed opposite to, or a little below the points of the fingers.



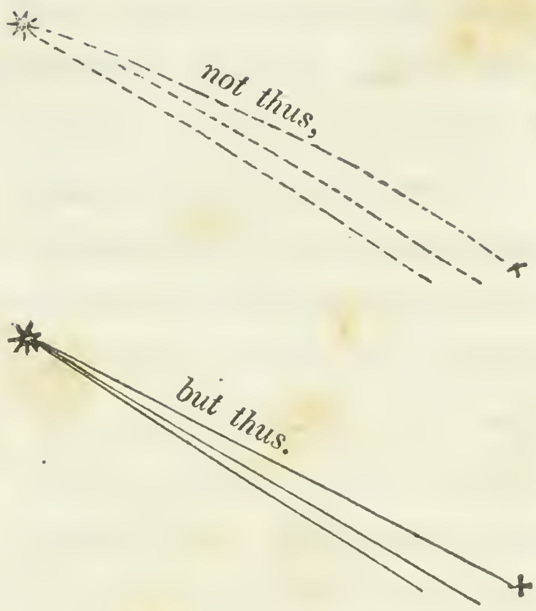
This position will enable the hand to move from left to right, and to draw curved lines with as much freedom in that direction, as from right to left.



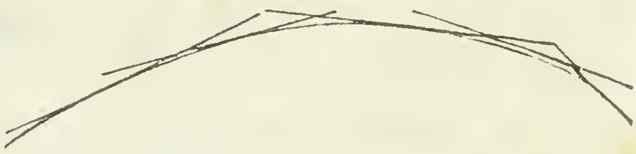
Let the learner now practise the drawing of such lines as are shown in the figure, from left to right.

In drawing straight lines by the hand, the learner should

not begin by timidly drawing dotted lines, as is usually done; but the pencil should be passed rapidly two or three times from one extremity of the line to the other, without touching the paper, and then the line should be drawn at one stroke. Should it not be correct, repeat the trial until it is right; after which, and not before, efface whatever is wrong.



In the same manner, curved lines may be first sketched out by drawing broken lines, and afterwards rounding off the angles and effacing the straight lines. These distinctions may appear trifling, and too minute, but nothing is more certain than that a careful and intelligent observance of them, will ensure a rapid and easy manner of sketching.

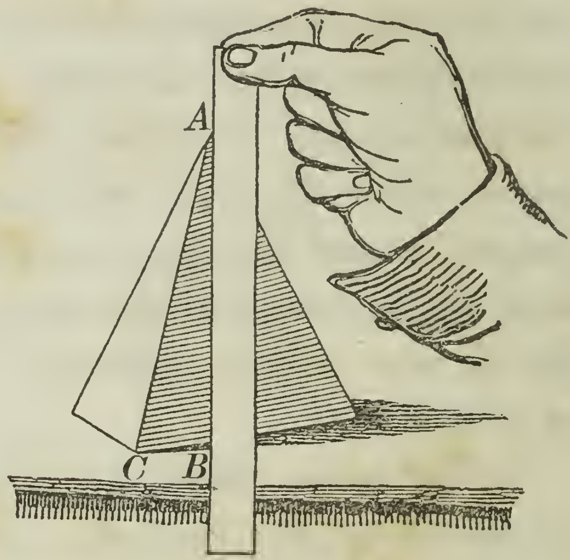


GENERAL REMARKS.

It is not intended, nor would it be possible, to give here more than a few practical hints concerning the general principles of the art of drawing. The learner, after familiarizing himself with them, and with the short directions as to the mechanical part, should copy some good drawings, under the direction of an instructor. He should then take some simple object, such as a book, a cup, an inkstand, &c., and placing it before him, endeavor to describe its position and proportions by means of its outline. This is done by comparing the lines which make up its outline with each

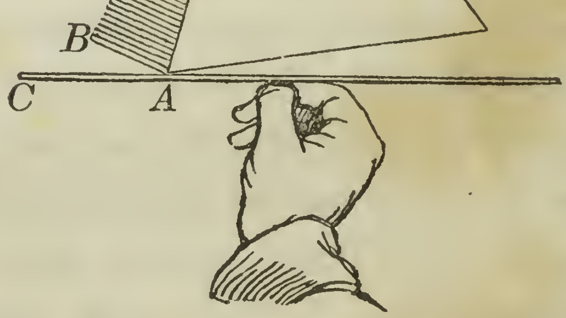
other, regarding both their comparative length and the angles which they make with each other. If the *direction* and *relative length* of each line are right, the drawing must be correct.

An easy help in finding the direction of a line nearly vertical, is to hold at arm's length, between the eye and object, (a pyramid, for example,) a ruler which serves as a plumb line. The edge of the ruler being vertical, when brought in range with the point *A*, will show how much the line *AC* varies from a perpendicular.



Now by drawing, or imagining to be drawn, a vertical line upon the paper, and then drawing a line making with it an angle equal to *BAC*, we shall have the direction of *AC*, or its inclination to a plumb-line *AB*.

To find the direction of a line nearly horizontal, we have but to balance the ruler, by placing its centre upon the thumb; then, continuing it in a horizontal position, and bringing it to range with the point *A*, we discover how much *AB* varies from a true level. Conceiving or drawing such an auxiliary level line upon the paper, and then laying down the angle *CAB*, we shall have the direction of *AB*, or its inclination to a horizontal line. This method is applicable



to the lines of distant objects, as well as to those which are near.

Having acquired by practice the power of sketching a single object in outline, the learner should place two or more objects before him, and endeavor, by means of drawing their outlines, to represent, in addition to their forms, their relative position with respect to each other. He should then proceed to shade them, and to draw the shadows which they cast upon each other, and upon the table or other surface on which they may be placed. The *colors* of the lights, shades, and shadows may then be added, and the representation will be complete.

SECTION II.

TOPOGRAPHICAL DRAWING.

1. *What is Topographical Drawing?*

Topographical Drawing is the art of representing upon a plane surface, the character and features of any piece of ground. Such drawings are always *plans*, and are distinguished from *geographical* maps by a greater degree of minuteness in their details. A system of signs has been universally agreed upon, and adopted; most of which, however, have a sufficient resemblance to the objects for which they stand, to enable them to be easily recognised.

The signs in the annexed plates have been adopted by the Engineer Department, and are used in all the plans and maps made by the U. S. Engineers.

These we shall proceed to explain, giving at the same time such hints as to the manner of drawing them, as may appear to be necessary.

The dimensions in which we represent such objects as houses, trees, roads, &c., in a topographical plan, depend, of course, upon the scale to which the drawing is made. Generally, for the sake of greater distinctness, they are enlarged to two or three times their proportionate size: unless the scale is very large, or when one of the objects of the plan is to exhibit every thing in its just proportion.

2. *Explain the figures on the next page.*

The figures in the first column explain themselves, in most cases, by some resemblance or appropriate sign; in other cases, they are purely conventional.

In Fig. 2, the signs of the plants are placed on the corners of squares drawn through the fields they occupy.

Fig. 3 shows the manner of expressing a pine forest with roads and the details of the leaves, in case the scale of the drawing will admit of their use. In forests, the trees are placed without any particular order or arrangement.

In Fig. 4, the horizontal lines, or the lines parallel to the top and bottom of the drawing, represent the watery portion of a fresh-water marsh: the rest of the figure, the earthy or *grassy* parts. In general, stagnant water is represented by horizontal lines; and meadow, or heath, by small tufts of grass. The combination of these two signs indicates morass, or wet ground.

Fig. 5 represents hillocks, or sloping ground. The paper is always left white to denote a *level;* and each one of the broken lines drawn from the summit to the base of a hill, indicates throughout its length the *direction* of the slope, or the line of *greatest* descent. The degree of blackness, or shade, produced by these lines shows the nature of the slope, from the perfect white of a level, to the deep blackness of an almost perpendicular descent.

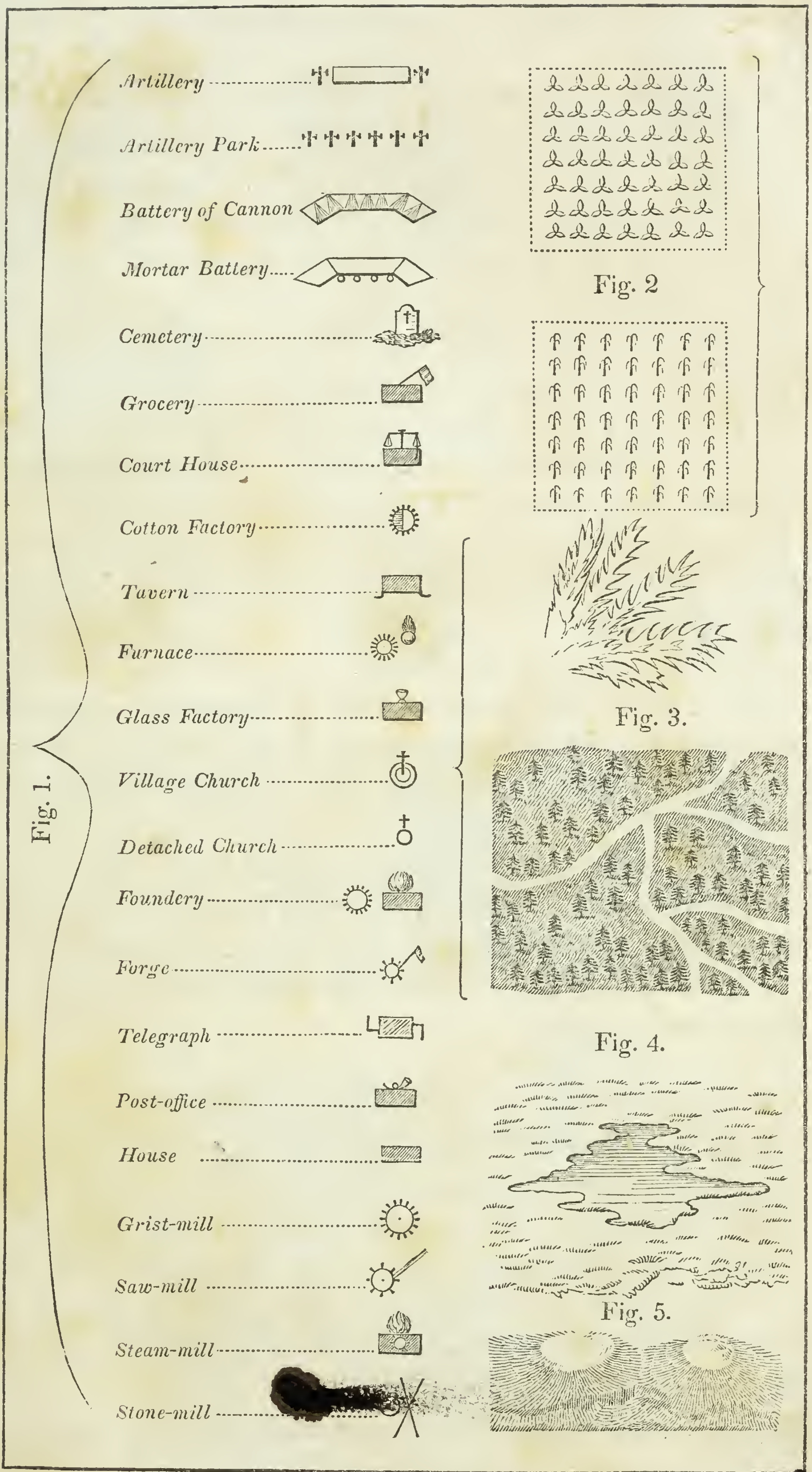


Fig. 1.

Fig. 2

Fig. 3.

Fig. 4.

Fig. 5.

3. *Explain the figures on the next page.*

Fig. 1 represents a rice-plantation; Fig. 2, an ornamental garden; Fig. 3, a cotton-field; Fig. 4, ploughed land, Fig. 5, an orchard; and Fig. 6, a vineyard. Figs. 1, 3, 5, and 6, are drawn as was described in the case of page 65, Fig. 2. Where it is not necessary to describe minutely the *kind* of crop existing upon the land, every kind of cultivation may be expressed as is done in Fig. 4.

Figs. 7, 8, and 9 indicate, respectively, the details of the leaves for oak, fruit, and chestnut trees, whenever their use in a plan is desirable.

Fig. 10 represents a heath and common road. It is left white, being a level, with the exception of the tufts of grass.

Fig. 11 is an oak, &c. forest.

Fig. 12 is a salt marsh. This is drawn in a different manner from a fresh-water marsh, being composed of unbroken horizontal lines, with tufts of grass interspersed among them.

Fig. 13 represents meadow, or bottom land, with a small stream running through it. The sign for the grass is here more regularly disposed than in a heath, or common.

Fig. 14 shows the mode of indicating different kinds of roads, fences, paths, &c.

4. *How is water represented?*

Running water, the water of lakes, and water that is affected by tides, are always represented by lines drawn within the outline, and parallel to the shores, in such a manner, that by gradually increasing the distance between the lines, which are at first very close together, the *shade* may be uniformly *lightened* from the shores to the middle. The course of the current is indicated by an arrow, with the head turned in the direction in which the water runs.

Fig. 1.

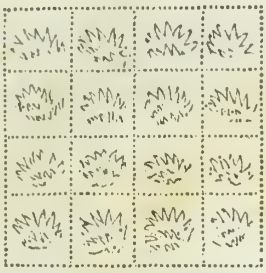


Fig. 2.

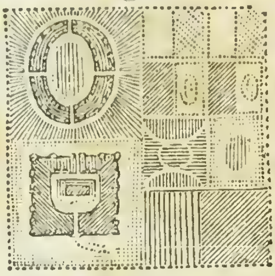


Fig. 3.

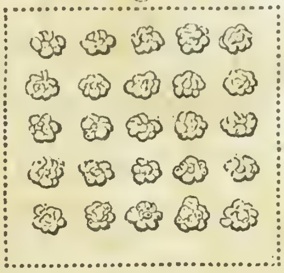


Fig. 4.

Fig. 5.

Fig. 6.

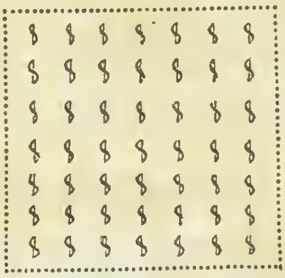


Fig. 7.

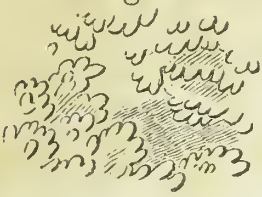


Fig. 8.

Fig. 9.

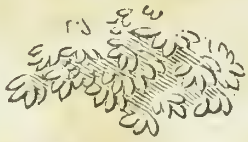


Fig. 10.

Fig. 11.

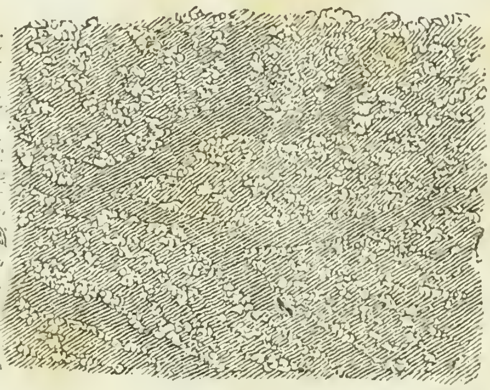


Fig. 12.

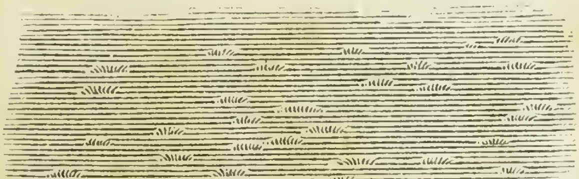
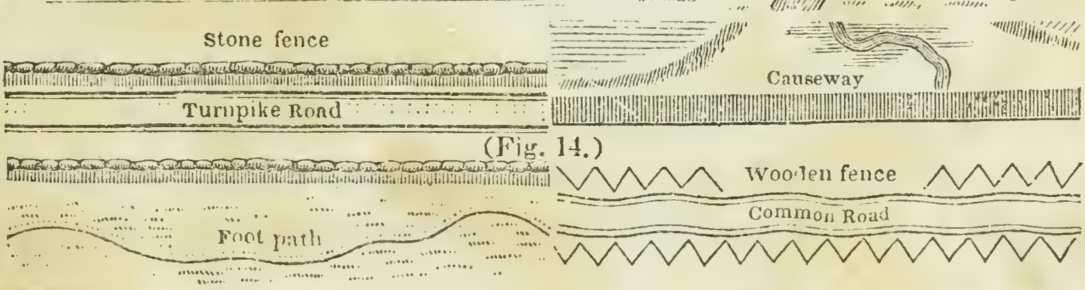


Fig. 13.



(Fig. 14.)

5. *Explain the figures on the next page.*

Fig. 1 represents the *rocky shore* of water thus shaded.

Fig. 2 denotes *rocks* that are above the surface of the water. Here, also, the lines indicate the direction of the descent from the highest point, near the middle, to the water line.

Fig. 3 shows the manner of representing *salt-works.*

Figs. 4, 5, and 6 show the three conditions of *sand-shoals.*

Fig. 7 is a sign used to show the direction of the *current.*

Fig. 8 shows that there is *no current.*

Fig. 9 indicates the different stages of the tides by means of *dots* introduced in all shading above low-water mark.

Fig. 10 represents rocks *sometimes bare*, and Fig. 11, sunken rocks.

Fig. 12 is a shore with sand-hillocks and fisheries.

Fig. 13 is a collection of signs used for describing the facilities or dangers of navigation.

Fig. 14 exhibits a river, with the different circumstances connected with its navigation, and the means of crossing it.

Fig. 15 is a *lake*, shaded in the manner before described.

In shading a piece of water by this method, this rule must be observed. Having drawn the outline, conduct the first shading line along *every shore*, (if there be more than one,) and around all islands, keeping it as close as possible to the shore-line.

When the first shading line is thus applied everywhere, take up the second one, laying it nearly as close to the first as the first is to the outline. When the second line is drawn wherever it can go, take up the third; increasing gradually and uniformly the distance between the lines, un-

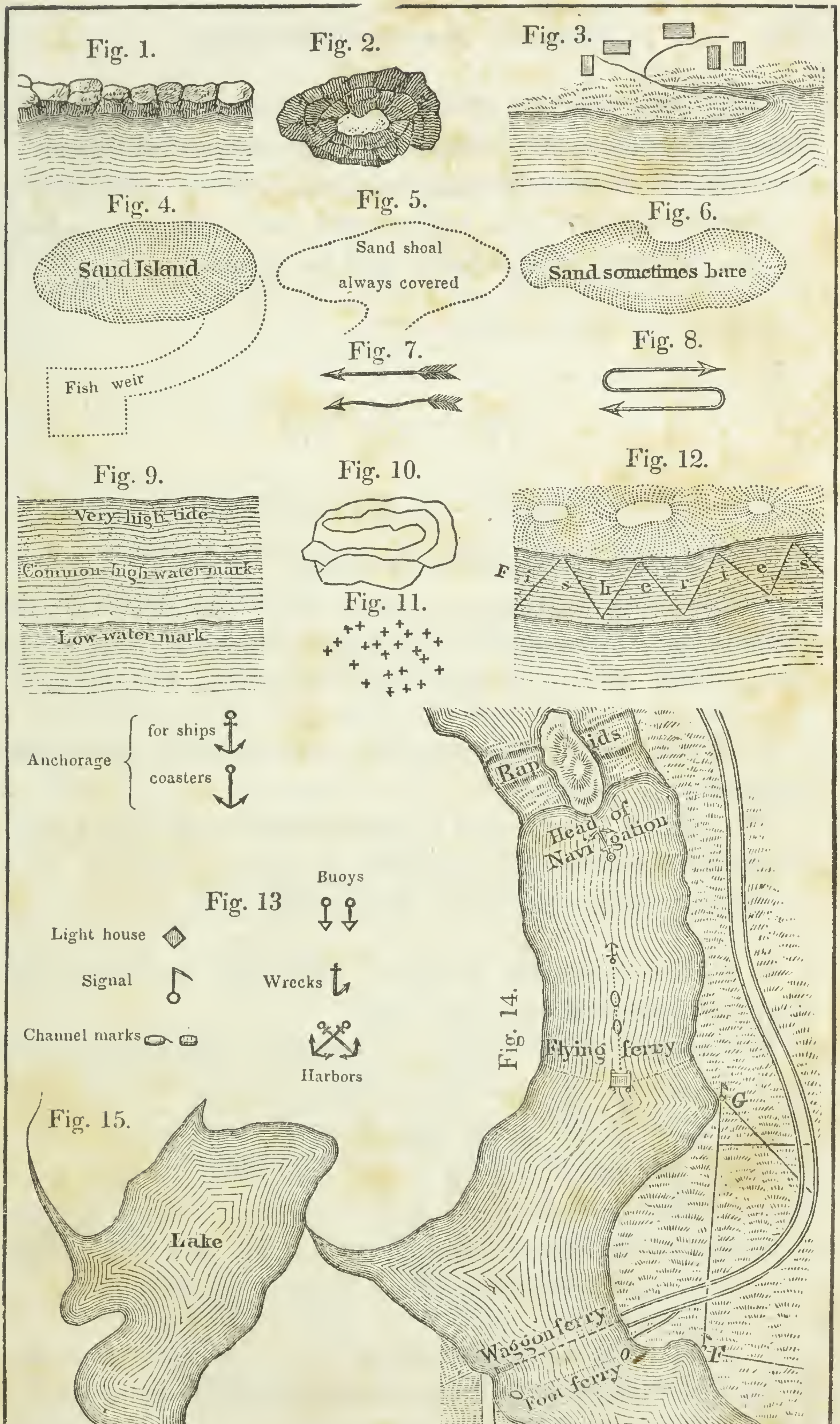


Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Sand Island
Fish weir
Fig. 5.
Sand shoal
always covered
Fig. 6.
Sand sometimes bare
Fig. 7.
Fig. 8.
Fig. 9.
Very high tide
Common high water mark
Low water mark
Fig. 10.
Fig. 11.
Fig. 12.
F i s h e r i e s
Anchorage
for ships
coasters
Fig. 13
Buoys
Light house
Signal
Wrecks
Channel marks
Harbors
Fig. 14.
Rapids
Head of Navigation
Flying ferry
G
Waggon ferry
Foot ferry
T
Fig. 15.
Lake

til they approach the middle, when it may be increased a little more rapidly, and the lines made somewhat thinner.

By pursuing this system, the shade will be graduated in a similar manner from every shore, and perfect symmetry in the positions of the lines will be insured.

SECTION III.

PRINCIPLES OF PLAN DRAWING.

1. *What are Geometrical Drawings?*

Geometrical drawings are those which are made for the purpose of conveying to the mind, through the eye, a just idea of the true proportions and dimensions of objects.

2. *What objects are generally represented in geometrical drawings?*

The objects represented in geometrical drawings are generally solid bodies, with irregular or curved surfaces, such as houses, blocks of wood, chairs, tables, &c.

3. *Can we generally conceive of their shape and dimensions from one single drawing or view?*

We cannot. For instance, if we place ourselves in front of a house, or opposite to one end of it, or if we stand behind it, or look down upon it from some great height, such as the top of a lofty steeple, we shall in each case have a different view of it; so that, unless we take different drawings of it, from several points, it will not be possible to convey any just notion of its general appearance.

4. *What is a horizontal plane?*

It is any plane parallel to the water-level, such as the level ground, the floor of a house, &c.

5. *What is a vertical plane?*

It is a plane perpendicular to a horizontal plane; such as the front or ends of a house, or the face of a vertical wall.

6. *How many kinds of geometrical drawings are necessary in order to represent the form and dimensions of an object?*

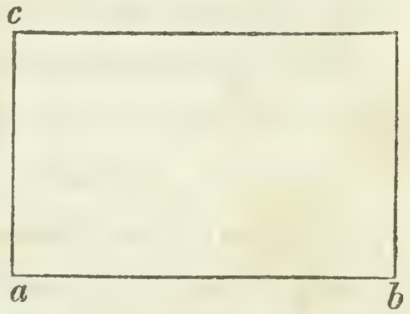
Three kinds only are necessary; viz., a PLAN, a SECTION, and an ELEVATION.

7. *What is a plan?*

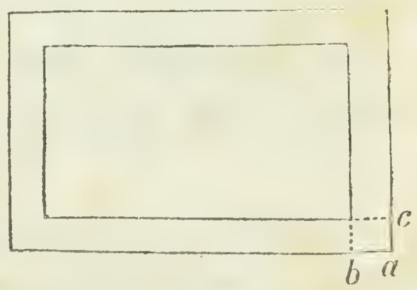
A plan of an object merely resembles the appearance which it would present to the eye, when viewed from a point directly above it.

In order to illustrate this more clearly, let us proceed to draw the plan of a small building

In commencing a building, the first thing necessary is to have a GENERAL PLAN, or plan of the foundation. Let us suppose that the building to be represented is a cottage, with a door and window only.

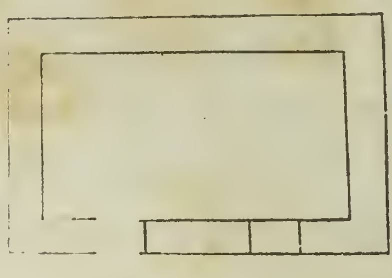


First, having fixed upon the scale on which the drawing is to be made, say 30 feet to the inch, lay off the length of the cottage 30 feet, on the line *ab*, and the width 24 feet, on *ac*, and complete the rectangle to represent the exterior dimensions of the cottage; that is to say, its length and breadth from out to out.



Next, lay off from the same scale the thickness of the wall from *a* to *b*, and from *a* to *c*, and draw the interior rectangle, having its sides parallel, respectively, to those of the outer one. This will represent the interior faces of the wall.

We see that this figure has nearly the same appearance as would be presented by the foundations of a small building, viewed from a point directly over them.



Doors and windows are generally marked in a ground plan. In order to distinguish them from each other, the lines of the foundation walls, which interfere with the doors, are rubbed out. The doors and windows will be marked accordingly.

The complete PLAN of the cottage is now drawn. It shows the size of the room, the thickness of the walls, and the width and position of the door and window.

By means of a plan, drawn according to a scale, it would be easy to lay out correctly, the foundations of a building and the doors and windows of the lower story. But after building a few courses, we should be obliged to stop for want of further directions, because the PLAN can neither explain the *height* of the doors or windows, nor the height of any other part of the building.

This proves what has already been stated, viz., that more than one kind of drawing of any object is always necessary in order to explain its form and dimensions. Before proceeding to the other kinds of geometrical drawings, mentioned above, we will add some further explanations and observations on the subject of PLANS.

8. The PLAN of any object is always supposed to be made on a horizontal plane or dead level. The necessity of following this rule will appear from the following considerations.

Suppose it were required to build a house on uneven ground, such, for example, as the side of a hill. Every one knows that in laying out the foundation, no reliance would be put on any oblique measurements made along the slope,

but that all the measurements would have to be made in horizontal lines. For instance, if you were to measure 30 feet obliquely, along the side of the hill, for the breadth of your proposed building, it would still be necessary to lay the first floor horizontally. After this was done, you might find the space which was laid out for the breadth of the building, reduced to 29 feet, to 28 feet, to 25 feet, or even to a less distance, according to the steepness of the slope of the hill. The plan of an uneven field, in which the dimensions were marked according to oblique measurements made upon the sloping or irregular surface of the ground, would therefore be of no use.

9. It is more difficult to draw the plan of any object having sloping or oblique lines, than to draw the plan of a building having only horizontal and vertical lines, because the oblique or sloping lines must all be reduced in a certain proportion.

10. The following are the rules for laying down truly, on a horizontal plane, the points and lines of all objects, any way situated, with respect to it.

11. The imaginary horizontal plane, on which the plan is made, and to which all points and lines are referred, is called the HORIZONTAL PLANE OF PROJECTION.

This plane may be so taken as to cut the object which is to be drawn upon it, or it may be taken directly above or below the object. But for learners, it is best to begin by supposing the horizontal plane to pass through the base, or lowest point of the given object.

In respect to such points of the object as stand upon the plane of projection, or coincide with it, there can be no difficulty, for such points are their own place or projections on the plane.

From every point without the plane of projection, a perpendicular is supposed to be drawn to the plane, and the point in which this perpendicular pierces the plane, will mark the true position of the point from which it was drawn.

If the plane of projection be supposed to lie below the given object, then all the points of the object will be above the plane of projection; and, consequently, all the perpendiculars, requisite for finding the position of these points on the plane of projection, will go downward from these points.

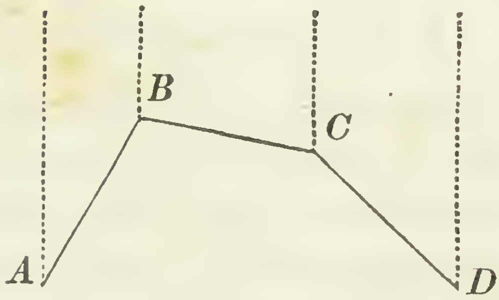
But if the plane of projection be supposed to be above the given object, then the several points of the object will be below the plane; and, consequently, all the perpendiculars, necessary for finding the position of these points on the plane of projection, will slope up from the given points.

12. Since the plane of projection, in plans, is always supposed to be horizontal, every perpendicular, whether dropped or raised, will be a vertical or plumb-line. Consequently, if we suppose two plummets to be suspended exactly over two points of an object, the plan of which is required to be drawn, the distance between the plumb-lines, measured perpendicularly, will be the true distance at which the two points ought to be laid down on the plan.

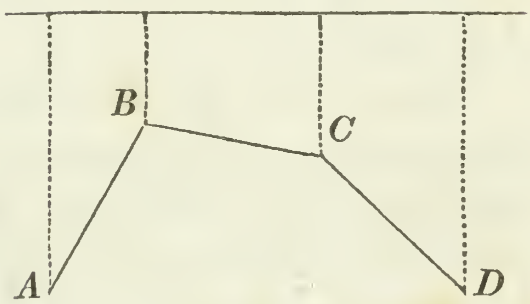
13. To explain this, draw three lines on the board connected together; all of the same length, but sloping unequally. These may represent the form of some sloping or oblique object, of which the plan is to be drawn.

The pupils will copy this and the following operations on their slates, without further directions, until the figure is completed. From the extremities of each of the three lines, draw dotted lines, parallel to each other, directed towards the top of the board.

These dotted lines may represent plumb-lines held over the various points of the oblique object. Now mark the various points of the oblique object by capital letters *A*, *B*, *C*, and *D*, from left to right.

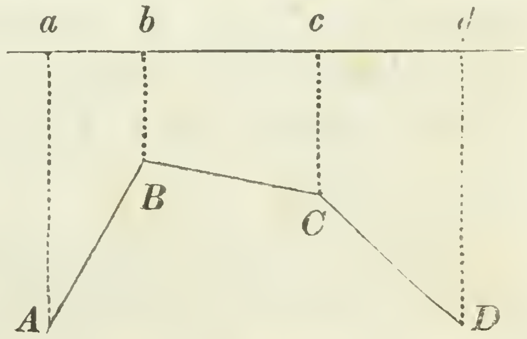


As the distances between the four plumb-lines, represented in the last figure, must be measured perpendicularly, not obliquely, draw a line above the given object, and perpendicular to the dotted lines, on which the said distances are to be measured.



At the points where the perpendiculars meet the horizontal line, make the letters *a*, *b*, *c*, and *d*, from left to right.

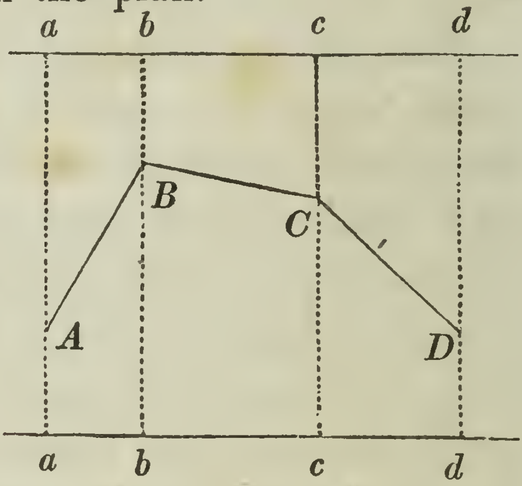
The distance between the points *a* and *b*, at the top of the figure, represents the exact distance between the plumb-lines suspended over the points *A* and *B*. Consequently, the perpendicular line *ab*, at the top of the figure, represents



the exact length which ought to be given to the oblique line AB, in drawing a plan of the given object.

The perpendicular line bc, at the top of the figure, in like manner, and for the same reason, represents the exact distance which ought to be given to the oblique line BC, in the plan of the object.

And the perpendicular cd, at the top of the figure, in like manner represents the exact distance which ought to be given to the oblique line CD, in the plan.



14. Let us now produce the dotted lines below the given object, and draw a second horizontal line intersecting them perpendicularly; and let us also mark the points of intersection by the same letters a, b, c, and d.

Then, since parallel lines are always at the same distance from each other, although produced ever so far, the distance between the points a and b, at the bottom of the figure, will be equal to the distance between the points a and b at the top; and the same for the distances between any other two points.

Consequently, the perpendicular distances ab, bc, and cd, at the bottom of the figure, will be equal to the perpendicular distances ab, bc, and cd, at the top; and, therefore, the lines ab, bc, and cd, at the bottom, will serve equally well to represent the respective lengths which ought to be given to the oblique lines AB, BC, and CD, in the plan of the given object.

Hence we see, that either the upper horizontal line ab, or the lower horizontal line ab, may represent the plane of projection, to be used in drawing the plan of the oblique object; the upper line will represent a plane passing above

the given object, and the lower line a plane passing below it.

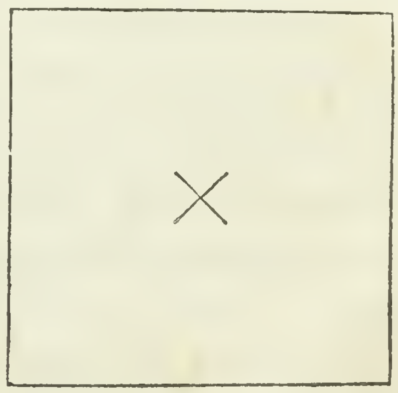
This illustrates what was before observed, that in drawing the plan of any object, it makes no difference whether the plane of projection is taken above or below it.

15. A line drawn from any point in a given object, and perpendicular to the plane of projection, is called the *projecting line of the point;* and the place where the perpendicular meets the plane, is called the *projection* of the point.

16. Let us illustrate the above rules by means of a square pyramid.—(Here let the teacher explain the shape of a square pyramid, and exhibit one to the class.)

If we look down upon a square pyramid, we shall see the extremities of its base, its vertex, and the four edges or oblique lines which are formed by the meeting of its sides. All these particulars must therefore be represented in the plan of a square pyramid.

The most convenient way, is to suppose the horizontal plane on which the plan is to be made, to pass through the base of the pyramid. For example, if we place the pyramid upon a table, the level surface of the table will represent the plane of projection. The base of the pyramid, now standing on the plane of projection, coincides with it, and will be its own projection, without any enlargement or diminution.

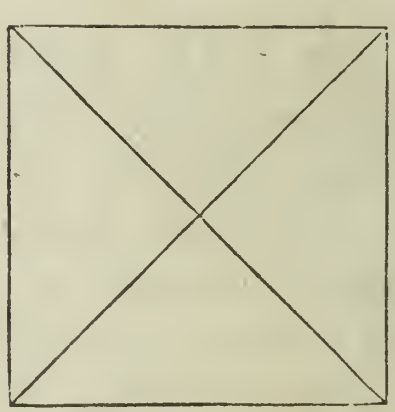


The base of the pyramid is a square. Represent it, therefore, on the paper or board, by drawing a square exactly equal to it.

In the present instance, the base of the pyramid coinciding with the plane of projection, and the pyramid being perfectly regular, it is evident

that a perpendicular dropped from the vertex, would fall exactly on the middle or central point of the base. Mark, therefore, the middle point of the square, and it will represent the projection of the vertex of the pyramid.

The four ridges, or oblique lines, remain to be drawn. But, one extremity of each of these lines passes through each angular point of the base, all of which are already marked on the plan. The other extremities of these lines all meet at the vertex of the pyramid, whose projection on the plan is also determined. Therefore, draw from the centre of the base four straight lines, one to each angle of the base, and they will represent, in the plan, the four ridges of the pyramid.



You see that the plan of the pyramid, now drawn upon paper, shows no dimensions but those of the base. It also indicates the particular point of the base over which the vertex stands; but it neither explains the height of the pyramid, nor the obliquity or slopes of its sides.

The plan, therefore, cannot alone explain the nature either of a building or pyramid, or of any other object, and recourse must be had to some other kind of drawings.

OF SECTIONS.

17. A section is a plane figure, formed by cutting any solid body into two parts. A solid body may be cut in a great number of directions: viz., horizontally, vertically, and obliquely: and hence, the number of sections which may be formed of any object, are infinite, or beyond calculation.

To avoid the confusion which might arise in plan-drawing, from sections taken at random, the geometrical drawing

called a section, is always taken vertically; that is to say, the object is supposed to be cut right down, perpendicularly, from top to bottom, by a vertical plane; in other words, it is supposed to be cut everywhere in a plumb-line.

A section is principally intended to show the heights of objects, and thereby to make up for the defects of the plan, which have already been explained.

Supposing it were required to measure the height of one of the sides of a room. This could not be correctly done by measuring diagonally or obliquely—that would be quite wrong. There is no way of finding the true height except by measuring vertically, or in the plumb-line.

If, then, we suppose a section of the room to be taken in which we now are, it is evident that if the section were taken in a sloping direction, it would cut the sides of the room obliquely. Such a section would therefore give an erroneous representation of the sides of the room.

Sections taken across any building or object, will of course serve to show the breadth as well as the height of its various parts. In order that this may be done truly, another rule must be laid down no less essential than the former: viz.,

In taking the section of any regular object, such as a rectangular building, the object is always supposed to be cut right across; that is, in a direction perpendicular to two opposite sides; and the same reason holds good in this case, which was given for employing a vertical plane.

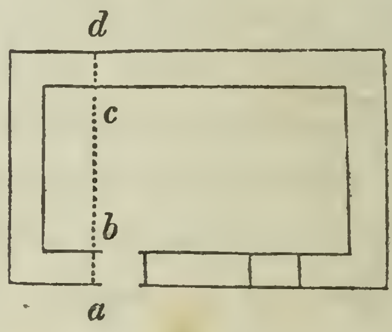
Supposing we wished to measure the breadth of this room. You see at once, that if we took the measurement obliquely, from angle to angle, the result would be quite wrong; and that there is no possible way of measuring the breadth of the room accurately, except in a direction perpendicular to its two opposite sides.

From these considerations it must be evident, that any

section of a building, or of an object, taken in a sloping or oblique direction, would not be of the smallest use, because it would either misrepresent the height, or the breadth, or both.

18. This being premised, let us now proceed to draw a section of the small cottage, of which we have already drawn the plan.

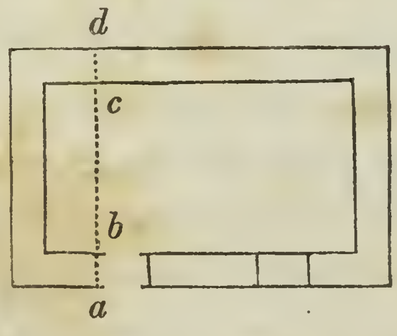
Let us suppose that the proposed section is required to pass through the door of the building. Draw a dotted line perpendicularly across the plan of the cottage, passing through the door. This dotted line will represent the direction in which the proposed section is to be taken.



Mark the points on the plan where the dotted line cuts the front and back walls of the cottage, by the letters *a*, *b*, *c*, and *d*. The distances between the points *a*, *b*, *c*, and *d*, show the breadth of the cottage and the thickness of the walls.

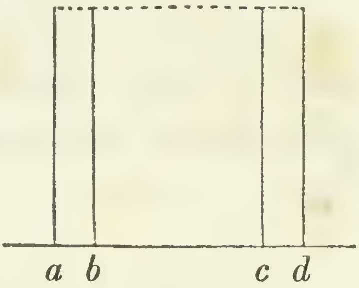
As the same dimensions which have been used in the plan must be again represented in the section, it will save time to transfer the whole of them, at once, from the plan to the section.

Therefore, draw a separate line to represent the level of the building, which will also be the ground line or base



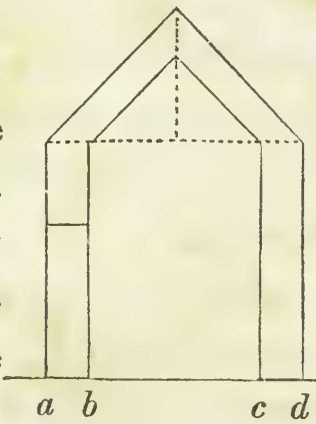
of the section. Then divide this line in the same manner as the dotted line *abcd* is divided in the plan.

Under the respective points of division, on this new line, mark the same letters *a*, *b*, *c*, and *d*. When this is done, the corresponding or like parts of both lines will be known by inspection.



From the points *a*, *b*, *c*, and *d*, on the ground line of the section, which represents the position and thickness of the walls of the cottage, raise perpendiculars to show the height of the walls. Join the tops of these perpendiculars by a dotted line which will be horizontal, and this line will show the level from whence the roof is supposed to spring.

The plan of the cottage is still supposed to remain on the board and slates, but is left out in some of the following figures. It will again be occasionally introduced, whenever it shall be necessary to point out the connection between the plan and the section.

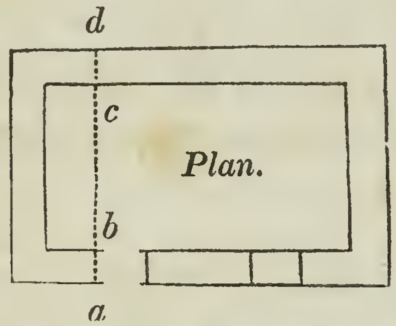


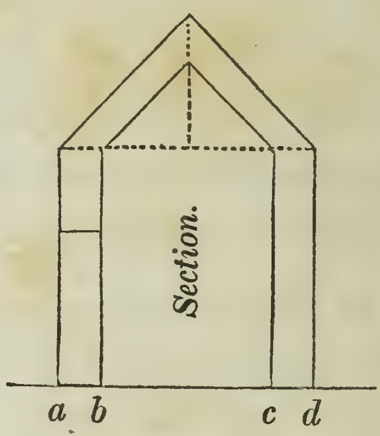
19. We will now suppose the roof to be a regular pitch roof. Therefore, bisect the last-drawn line, in order to find the middle of the building; and from the point of bisection raise a perpendicular, to show the height of the roof. From the extremities of this perpendicular, draw an oblique line to the outside of the top of each wall: this will show the sides of the roof. Then draw right lines interiorly, parallel to the last lines, to show the thickness of the roof.

As the section is supposed to pass through the door of the cottage, a line must be drawn to represent the top of the door, and to show the height of it.

The section which has just been drawn, is only intended to give a general notion of this kind of geometrical drawing.

Many particulars are therefore omitted, which it would be proper to introduce into a finished section of a building. For instance, the depth and thickness of the foundation, the recess of the door, the thickness of the rafters and other parts of the roof; also, its projection over the walls, if formed with eaves. These, and other details might easily have been represented, by adding a few more lines. The





rough section of the cottage is now complete, and you may observe, that those dimensions which are marked with the same letters, agree in both.

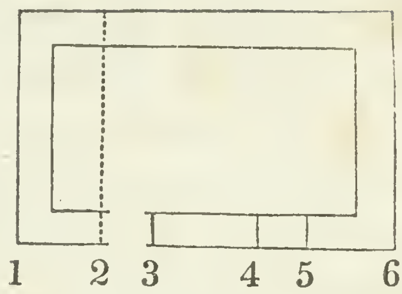
The plan and sections, as they stand at present, explain sufficiently the general dimensions of the cottage, and the proportions of the roof and door; but they do not show the height of the windows, nor the general appearance of the building.

The latter particulars cannot be represented without the assistance of the third kind of geometrical drawing, before mentioned, called an ELEVATION.

OF THE ELEVATION.

20. An ELEVATION is the view of any upright side of a building or other object, nearly such as it would appear to a person standing directly in front of it.

In order to understand this definition more clearly, let us draw an elevation of the front of the cottage.



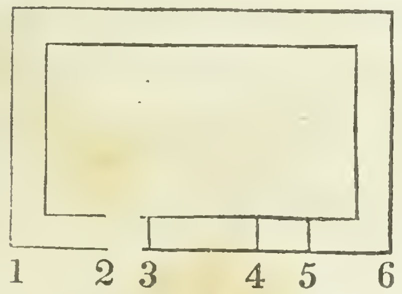
As the principal dimensions of the front of the cottage appear in the plan, let the various points be marked by the figures 1, 2, 3, 4, 5, and 6.

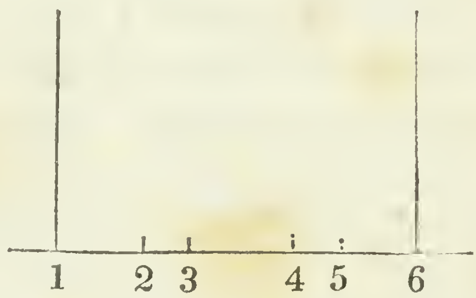
The points thus marked show the length of the front of the cottage, and the breadth and position of the door and window.

As all these dimensions must appear in the elevation of the cottage, the easiest method will be to transfer them from the plan to the elevation at once.

Draw, therefore, a separate line, to represent the ground line, or level upon which the front of the cottage stands; and upon this line, set off a distance equal to the length of the cottage, and divide it in the same manner as the front of the cottage is divided in the plan.

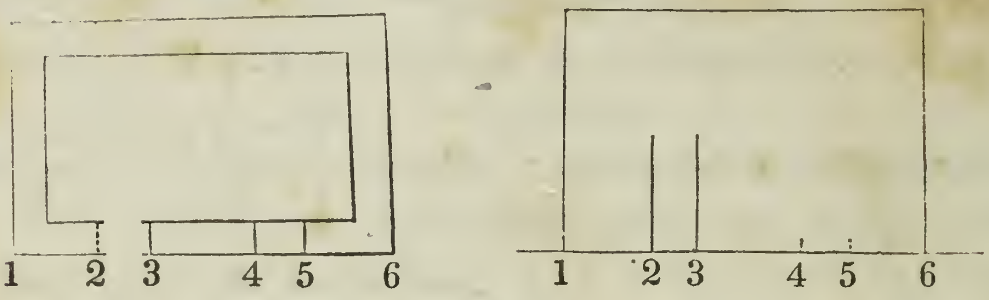
Mark also the various points of division on this new line, by the figures 1, 2, 3, 4, 5, and 6. When this is done, the



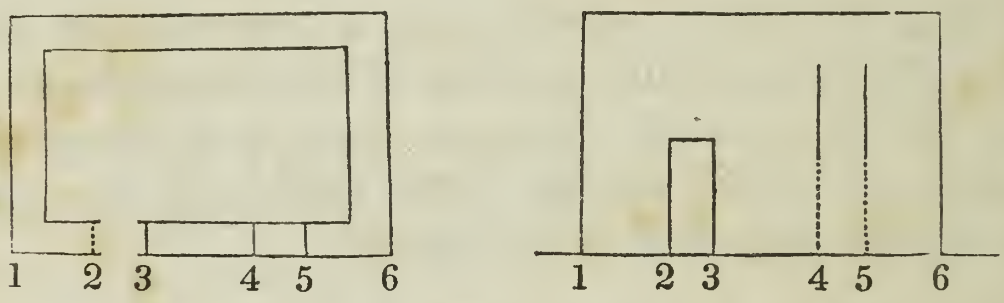


corresponding or equal parts in the plan and in the ground line of the elevation, are known by inspection. From the points 1 and 6 of the ground line of the elevation, let perpendiculars be drawn to show the height of the walls. Now, since the height of the walls is already represented in the section, take that height in the dividers and lay it off on the perpendiculars through 1 and 6.

Join the top of these perpendiculars by a straight line. This line will represent the bottom of the roof of the cottage.

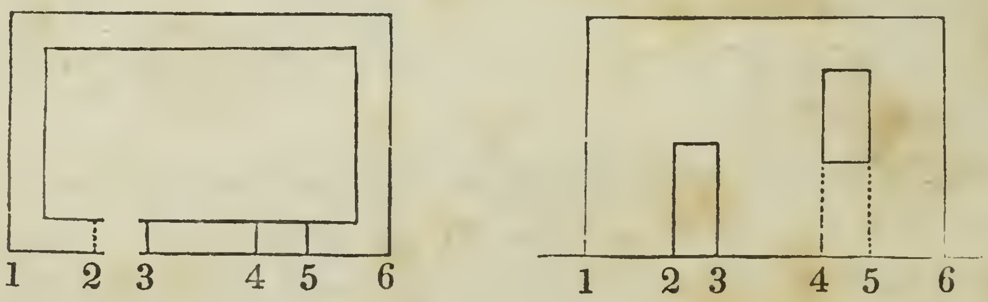


From the points 2 and 3 of the ground line of the elevation, which represent the width of the door, raise perpendiculars to show the height of the door. Find the proper length of these perpendiculars by measuring the height of



the door in the section, and then transfer it to the elevation. Complete the form of the door by joining the top of the above perpendiculars.

From the points 4 and 5 in the elevation, which mark the position of the window, raise perpendiculars to find the sides of the window. Next complete the window by draw-

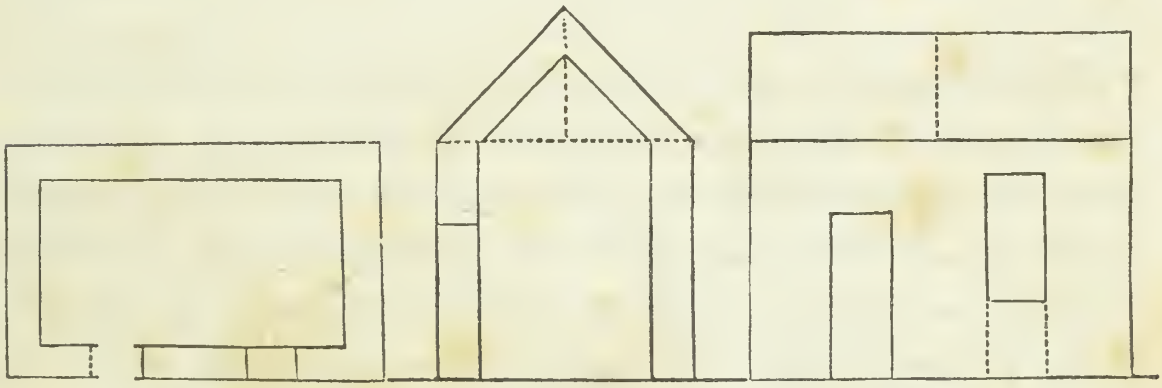


ing the top and bottom of it, at their proper height. Dot that part of each of the last perpendiculars, which falls below the bottoms of your windows.

The form of the roof is now alone wanting. The length of the roof must of course be equal to the length of the building, and the height of it may be found by referring to the section.

It is a general rule, in geometrical elevations, never to represent the height of any sloping object by oblique measurements taken along the slope; but, by dropping a perpendicular from the highest point, or vertex of the slope, to the level of the lowest point or base of it.

In short, the height of any sloping object in a geometrical elevation is measured by that perpendicular line, which would be called the altitude of any similar figure or body, in Geometry. Therefore, in transferring the height of the roof from the section to the elevation, make it in the elevation equal to the dotted perpendicular drawn in the section. Next draw the roofs: when this is done, the drawings of the cottage are as follows:



Plan. *Section.* *Elevation.*

21. The Plan, Section, and Elevation of a small cottage are now complete, and from these three geometrical drawings put together, every dimension necessary for explaining the proportions of the building may be known.

The length of the building is shown in the plan and elevation, and is the same in both.

The breadth of the building, and thickness of the walls,

are shown in the plan and section, and are the same in both.

The breadth of the door, and that of the window, are shown in the plan and elevation.

The height of the door is shown in the section and elevation, and is the same in both.

The height of the window is shown in the elevation only. But if the section had been taken through the window, instead of the door, then the height of the window would have been shown, and not that of the door.

The height of the walls, and the perpendicular height of the roof, are shown in the section and elevation, and are equal in both. But the particular form of the roof is clearly explained in the section only.

REMARKS ON ELEVATIONS.

22. An elevation is always supposed to be drawn on a vertical plane, which is called the *vertical plane of projection.*

Those points of an object which lie in, or coincide with, the plane on which the elevation is drawn, are their own projections on that plane. Those points of the given object which lie without the plane of projection, must be transferred to it, by lines drawn from the points and perpendicular to the plane of projection. Such lines are called *projecting lines.*

Since all the projecting lines which determine an elevation are perpendicular to a vertical plane, they must necessarily be horizontal. The walls or sides of a building are vertical planes, being built according to a plumb-line ; and therefore, in taking a geometrical elevation, the plane on which it is made may be supposed to coincide with the front of the building, or any other side which is to be represented.

When this is done, the length and height of the side of the building, and the height and breadth of the doors and windows, &c., may be laid down in a geometrical elevation, according to their actual dimensions from measurement.

The roof, from its sloping figure, is the only part of the exterior side of the building which cannot agree with the plane of projection; and hence, in drawing the elevation of the cottage, it was necessary to diminish the oblique lines of the slope of the roof, in order to find the true vertical height of it. They were diminished in the same way that the oblique lines are diminished in a plan, in order to find the base of any slope.

23. It is not necessary, in a geometrical elevation, that the plane of projection should be supposed to agree exactly with the upright side of the building or object which is to be represented. But when they do not agree, it is necessary that the plane of projection should be parallel to the upright side of the building or object, of which the elevation is to be drawn. In that case, the projecting perpendicular will form, on the plane of projection, a figure exactly similar to the front of the building or object. Consequently, if you suppose a plane of projection to be chosen, parallel to the upright side of a building or other object, of which an elevation is required, then the dimensions of the various parts of the upright side of the given object may be laid down in the drawing in their true proportions, according to measurement.

24. From the figures which have been drawn, and the instructions which have been given, on the subject of Plan-drawing, it appears that plans and elevations are drawn exactly according to the same principles, with only this difference: that in a plan, the plane of projection is always

horizontal, whereas, in an elevation, it is always vertical.

All horizontal planes, which may be used as planes of projection for drawing the plan of a building, will be parallel to each other; but the vertical planes, on which the elevations are drawn, may be oblique, perpendicular, or parallel to each other.

For example, the several floors of any building, being all level, and all the points of each at the same height from the ground, are horizontal planes parallel to each other. But of the walls of a building, which are all vertical planes, some two of them may be perpendicular to each other, such as the side and end walls; while others may be oblique to each other, as is often seen in irregular buildings.

OF OBLIQUE ELEVATIONS.

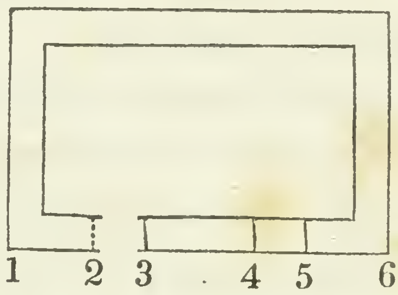
25. If, in drawing the elevation of any rectangular building, the plane of projection were chosen oblique to one of the sides, instead of parallel to it; then, the length of that side of the building and the breadth of the doors and windows would be diminished in the drawing, in such a manner as to give a false notion of the object. In an oblique elevation of this kind, the projecting lines which are drawn perpendicular to the plane of projection, will be oblique to the building; and hence, all the dimensions except those which are vertical would be diminished or misrepresented in the drawing: hence, such elevations are of little use, and are therefore seldom made.

But, although oblique elevations of the fronts of buildings are seldom made, it often happens that the front of a fine building is ornamented with columns, mouldings, and architectural decorations, many parts of which are oblique to the general plane of the front of the building, beyond which they project.

The proper methods of representing such ornaments in geometrical elevations, cannot therefore be well understood, unless the principle, according to which oblique elevations of any upright object may be drawn, is clearly explained.

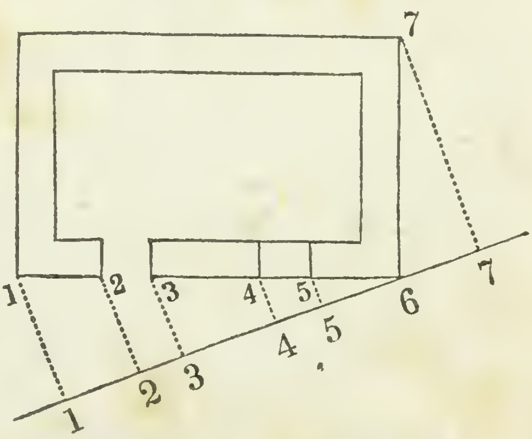
This being premised, we shall give an example of the method of drawing an oblique elevation of the cottage, of which we have already drawn the plan, and section, and geometrical elevation.

26. Resume the plan before drawn, and mark thereon the points 1, 2, 3, 4, 5, and 6.



A straight line must next be drawn, to represent the new plane of projection, on which the oblique elevation is to be made. This new plane of projection may either be supposed to coincide with some line of the front face of the building or not. We shall take it to coincide or agree with that extremity of the front of the building which is marked by the figure 6.

Draw, therefore, a right line through the point 6, forming an acute angle with the front of the building, and this line will represent the new plane of projection, which is vertical.



From the various points of the front of the building, draw perpendiculars to the last line, and dot them. These perpendiculars will determine the true places of the points in the oblique elevation.

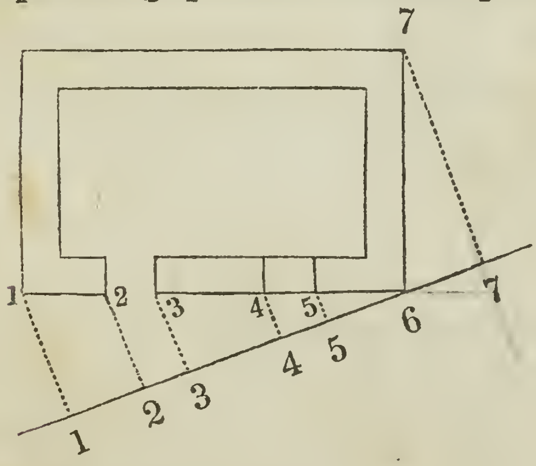
Mark, therefore, in like manner, by the figures 1, 2, 3, 4, 5, and 6, the several corresponding points on that line which represents the plane of projection.

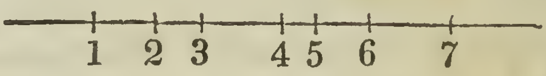
That end of the cottage which is nearest to the plane of projection must also be represented. One extremity of it coincides with the said plane. From the other extremity draw a dotted perpendicular to the plane of projection, and mark the corresponding points, at the ends of this line, by the figure 7.

The distance between the points 1 and 6 in the plan, shows the length of the front of the cottage; and therefore the distance between the corresponding points 1 and 6, on the plane of projection, will also represent the length which ought to be given to the front of the cottage in the oblique elevation.

The distance between the points 6 and 7, in the plan, represents one end of the cottage; and therefore the distance between the corresponding points 6 and 7, on the plane of projection, will also represent the length which ought to be given to that end of the cottage in the oblique elevation.

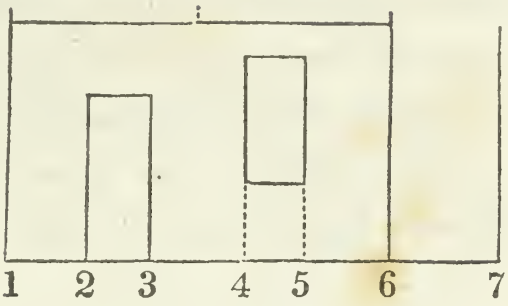
And, in like manner, as the breadths of the door and window are represented, respectively, by a certain distance in the plan; so the same dimensions, in the oblique elevation, must be represented by the distance between the corresponding points, on the plane of projection.





You will, therefore, draw a line for the ground line of the oblique elevation. Divide this line in the same manner as the one which represents the plane of projection, and

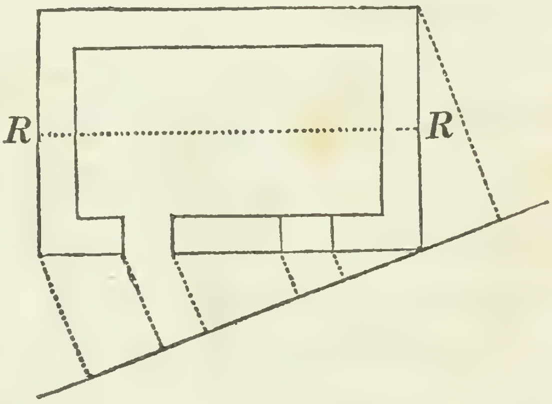
mark it with the same numeral figures. From the points 1, 6, and 7, on the ground line of the elevation, raise perpendiculars equal to the height of the cottage ; and draw the upper line. At 2 and 3 also draw perpendiculars, and lay off the height of the door ; and do the same at 4 and 5 for the window—dotting those parts of the perpendiculars which lie below the window.



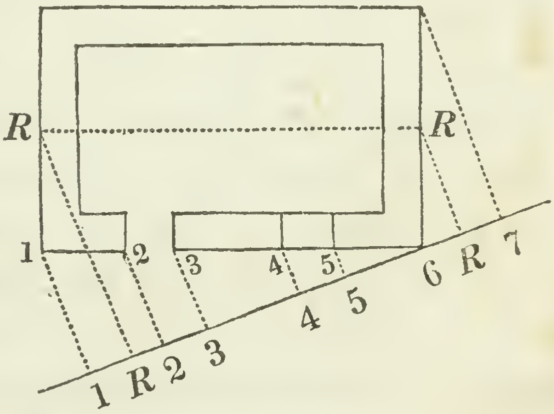
The roof only remains to be drawn. Before this can be done, it will be necessary to find the points where the ridge ought to be laid down in the plane of projection.

The ground plan of the cottage does not show the ridge of the roof ; but it is evident that the ridge of a regular roof with a simple pitch, must be directly over the middle of the building.

In order to save the trouble of drawing a separate plan, draw a dotted line *RR* through the middle of the plan already drawn, to represent the ridge.

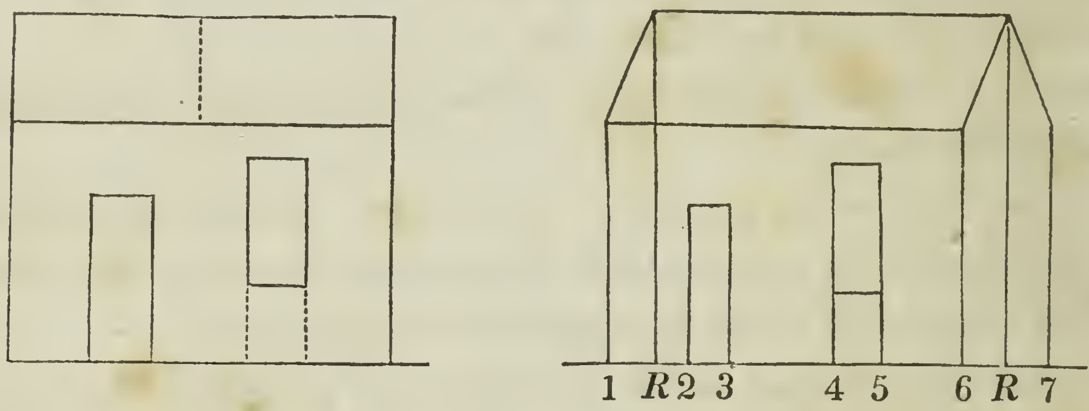


From the points *R* and *R*, which represent the extremities of the ridge of the roof, draw dotted perpendiculars to the plane of projection, and mark the points where they meet the plane, by the letters *R* and *R*. The points *R* and *R* must next be transferred to the ground line of the oblique elevation. From these new points *R* and *R*, draw perpendiculars and



lay off the height of the cottage, which is found by referring to the section: then draw the upper line, which will represent the ridge; after which, draw oblique lines from the extremities of the ridge to the proper points, in order to complete the form of the roof.

The oblique elevation of the cottage is now finished, as below, where the parallel elevation is also given.



The heights of the various parts of the oblique elevation agree with those of the section and front elevation; but all the other dimensions are changed, being less than they were in the plan.

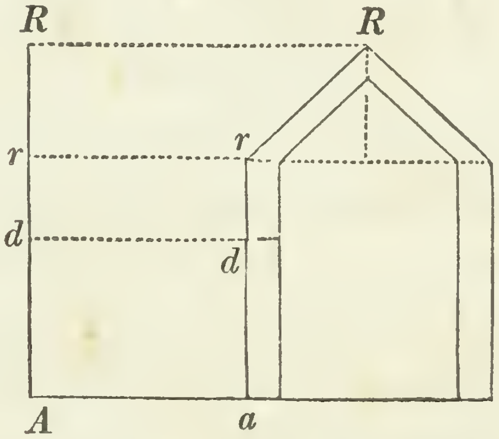
If the plane on which the oblique elevation was made had formed a greater angle with the front of the building, then the various dimensions, in that part of the oblique elevation which represents the front of the cottage, would have been still more diminished.

In the direct elevation of the front of the cottage, it was not necessary to take any notice of the points *R* and *R*, because they fell directly over the ends of the building. The two ends of the building being perpendicular to the plane of projection, will fall in the parts of the vertical lines through *R* and *R*, which lie between the ridge *RR* and the upper line of the front.

27. In transferring the several heights from the section to the elevations, each dimension was measured separately,

one after the other; but it is best to transfer the various heights from a section to an elevation all at once, in the same manner as the dimensions are transferred from the plan to the elevation.

Remember that the section of the cottage was formed by a plane cutting it through the door, and perpendicular to the front, and that *ar* is the line in which such plane cuts the front. At a convenient distance from the section draw the vertical line *AR*. Then, through the various points of the section whose heights you

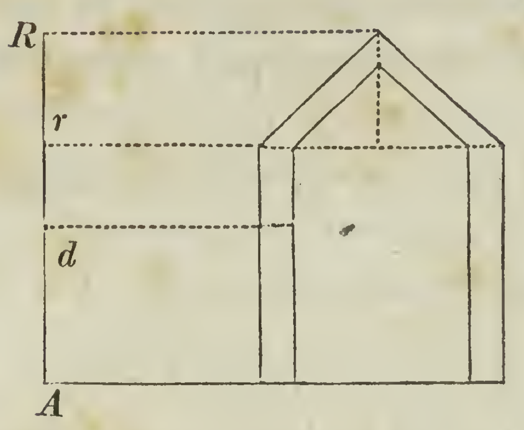


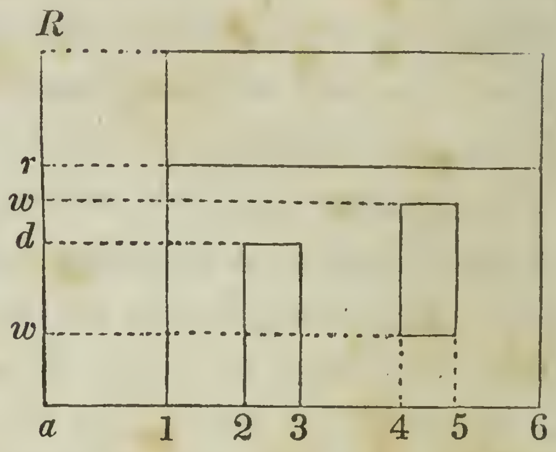
wish to note, draw the dotted horizontal lines *RR*, *rr*, *dd*, and *Aa*, and note the points in which they cut the vertical line *RA*.

The distance between the two points *a* and *d* in the section, represents the height of the door in the cottage; and the distance between the two corresponding points *A* and *d*, on the vertical line *AR*, will represent the height which ought to be given to the door in a geometrical elevation; and the same for all other points. Hence, all the points necessary for transferring the several heights of the front of the building from a section to an elevation, are now marked on the vertical line *AR*.

28. If you wish to draw the front elevation from the section and plan, draw a line to represent the ground line. Then draw a vertical line *AR*, and make it equal to *ar*, which shows the height of the cottage; and lay off in the same manner the height of the door *ad*, and the height of the wall *ar*. Then, at a convenient distance from *a*, mark the corner of the cottage 1; and from the plan lay off the dis-

tances from 1 to 2, 1 to 3, 4, 5, and 6; and through 2, 3,

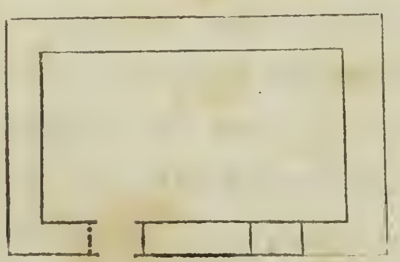




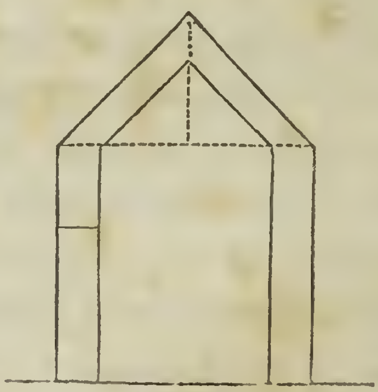
and 6 draw perpendiculars, which being met by parallels through *d*, *r*, and *R*, will determine all the parts of the cottage. The two heights *aw*, *aw*, to the bottom and top of the window, are not found in the section, but are taken from the oblique elevation.

29. After finishing an elevation, or other geometrical drawing, the superfluous or dotted lines representing planes of projection, scales of heights, &c., are rubbed out; excepting only those imaginary lines, marked in the plan, which show the direction according to which the sections or oblique elevations accompanying the plan may have been taken.

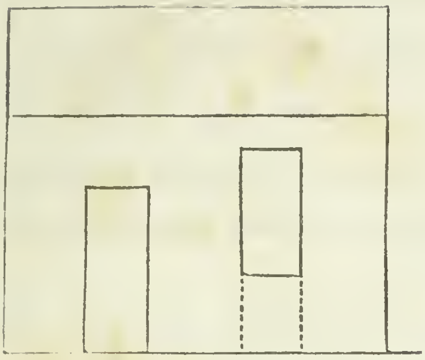
Let us, therefore, rub out the superfluous lines, letters, &c., in the figures which have been drawn, leaving only such as are necessary to explain the connection between the plan and section, and between the plan and oblique elevation. We shall then have—



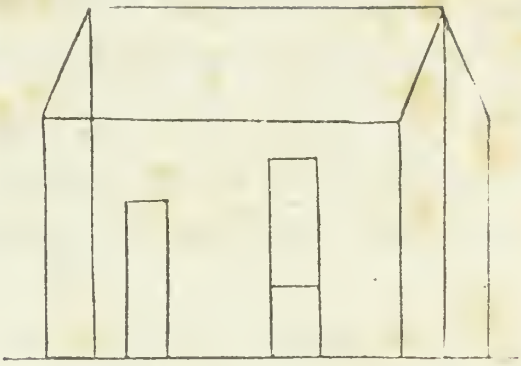
Plan.



Section.



Front Elevation.



Oblique Elevation.

The Plan, Section, and Elevation, completed above, are sufficient to give a full insight into the principles of plan drawing.

GENERAL REMARKS.

30. All plans, sections, and elevations are drawn by laying down a certain number of points and lines truly, on some plane surface, according to geometrical principles. In drawing some objects, it may be necessary to lay down a great number of points and lines: in others, only a few; but whether the number be great or small, each individual point or line must be drawn, in all cases, according to some one or other of the foregoing rules.

Plans which, as before stated, resemble the appearance of any object viewed from a height directly above it, do not carry a very just notion of the object to persons ignorant of the principles of plan drawing; because opportunities of looking perpendicularly or directly down upon objects are not common.

Ground plans, or foundation plans of buildings or other works, do not give any just notion of the appearance of the object represented; because when a building is finished, it is impossible, from any point of view whatever, to see the various walls and foundations, in the manner in which they must be represented in the ground plan, the whole of these parts being hidden by the roof. In fact, the ground

plan of any finished building is, properly speaking, a horizontal section through the various walls—the only difference between it and the common section consisting in this, that the common section is taken vertically, whereas the section which exhibits the ground plan is taken horizontally.

31. In plans, sections, and elevations of any object, when the various points and lines have been laid down according to the rules of projection, it is usual afterwards to color or shade the figure in order to make a finished drawing.

The art of plan drawing, therefore, comprehends two distinct operations: first, the projection of the lines which form the representation of the object; and secondly, the shading or coloring of it.

In colored plans and sections, masonry is generally made red; wood so as to represent its own natural color; earth of a sandy color; iron of a dark blue; and water of a lightish blue.

In plans not colored, masonry is generally made dark, while wood and other substances are made lighter.

In sections not colored, different substances are shaded darker or lighter, according to the fancy of the draughtsman.

In plans of buildings, the doors and windows are left blank, while the walls are either colored or shaded. And in sections, a marked distinction of color or shade is also made between the solid part of the walls, and the doors, windows, or other apertures which may be represented.

In elevations of any object, whether colored or not, the various parts are shaded in such a manner as to resemble, as much as possible, the outward appearance of the object.

BOOK IV.

SECTION I.

OF ARCHITECTURE.

1. *What is Architecture?*

Architecture is the art of construction.

2. *Into how many branches is it divided?*

Into three principal parts:—

1st. Civil architecture, which embraces the construction of public and private edifices.

2d. Naval architecture, which embraces the construction of vessels, ports, artificial harbors, &c.; and

3d. Military architecture, which embraces the construction of forts, redoubts, and all military defences. We shall speak here only of civil architecture.

3. *What are the elements of architecture?*

They are the MOULDINGS.

4. *What are mouldings?*

They are the projecting parts which serve to ornament architecture.

5. *How many kinds of mouldings are there?*

Three kinds: those bounded by planes; those bounded by curved surfaces; and those bounded by both plane and curved surfaces.

6. *What are the principal plane mouldings?*

They are the Fillet, the Drip, and the Plate-band.

7. *What is a fillet?*

It is a square moulding which projects over a distance equal to its height.

8. *What is a drip?*

It is a large projecting moulding, hollowed on the under side, and placed in cornices to protect the edifice from rain

9. *What is a plate-band?*

It is a large and flat moulding which projects but little.

10. *What are the principal circular mouldings?*

The Ovolo, the bead or Astragal, the Torus, the Cavetto, the Scotia, the Cyma-recta, the Cyma-reversa, and the Ogee.

11. *What is an Ovolo, and how do you trace it?*

An ovolo is a moulding flat on the top and bottom, and whose circular projection is equal to its height.

To describe it, make the perpendicular height AD equal to the projection AC: then, with A as a centre, describe the arc DC. If you wish to make a flattened ovolo, with B as a centre and BA as radius, describe an arc: then, with A as a centre and AB as a radius, describe a second arc, meeting the first in C. Then, with C as a centre, describe the arc BA.

12. *What is the Bead or Astragal, and how is it traced?*

It is a thin moulding, of which the circular projection is equal to half the height. To trace it, describe a semi-circumference, of which the diameter AB represents the height of the moulding.

13. *What is a Torus, and how traced?*

It is a moulding similar to the bead, but thicker. It is

traced by describing a semi-circumference on the height *AB* as a diameter.

14. *What is a Cavetto, and how is it traced?*

A cavetto is an ovolo, of which the centre *C* is in a perpendicular from the extreme projection of the moulding. It is traced by describing the quarter of a circumference from *C* as a centre. The second figure presents a cavetto reversed.

15. *What is a Scotia, and how is it traced?*

It is a hollow moulding, formed by several cavettos with different centres. The second figure represents a reversed scotia. The circular parts are described with the centres *A* and *B*.

16. *What is a Cyma-recta, and how is it traced?*

The cyma-recta is composed of an ovolo and a cavetto. To describe it, draw the line *AB*, and then divide the projection of the moulding into two equal parts by the perpendicular *CD*, and produce the line *B*: the point *D* will be the centre of the ovolo, and the point *C* of the cavetto, which together form the cyma-recta.

The flattened cyma-recta is a similar moulding. To trace it, it is necessary, after having divided the line *AB* into two equal parts, to construct an equilateral triangle on each of the parts. The points *C* and *D* will then bè the centres of the arcs which form the moulding.

17. *What is the Ogee, and how is it traced?*

The ogee is a moulding composed of the same parts as the talon, but differently placed. Having joined the points *A* and *B*, we draw through the middle point of this line the line *CD*, parallel to the fillets *A* and *B*, and the points *C* and *D*, in which it meets the perpendiculars, are the centres of the arcs which form the moulding. If the ogee is flattened, the centres are the vertices *A* and *B* of the equilateral triangles, each constructed on the half of *DC*.

18. *How do you trace this moulding, when its projection exceeds its height?*

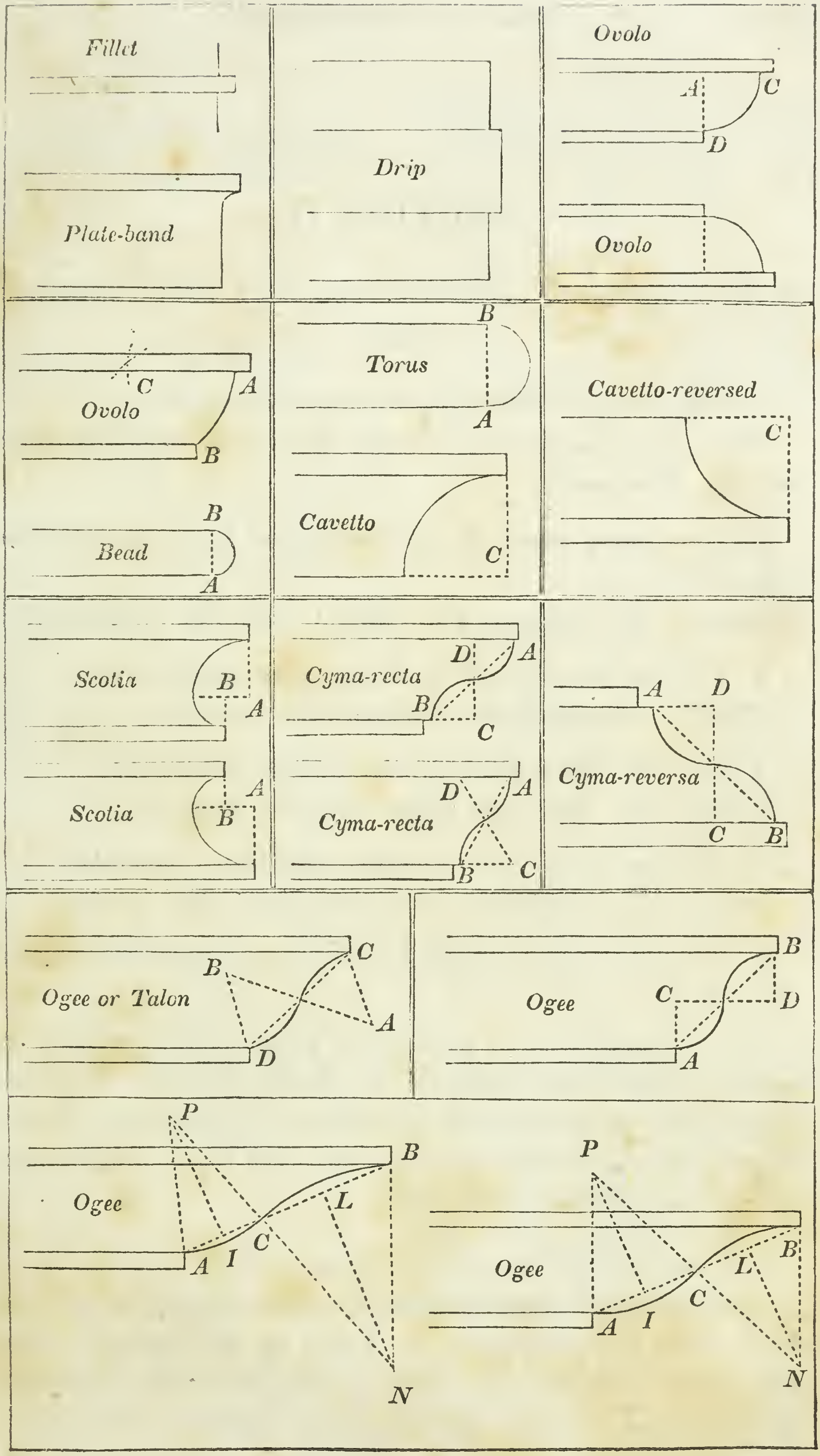
Having joined the points A and B, divide it into two equal parts AC, BC, and then draw IP perpendicular to CA at the middle point. Next, draw LN perpendicular to B at the middle point, but in a contrary direction. Then draw BN perpendicular to the fillet; after which draw NC, and produce it to P: then P and N will be the centres of the arcs. To give grace to this moulding, the part BC is sometimes made shorter than the part CA: in every other respect the construction is the same.

19. *How are these mouldings to be used in combination?*

They are not to be used at hazard, each having a particular situation to which it is adapted, and where it must always be placed. Thus, the ovolo and talon, from their peculiar form, seem designed to support other important mouldings; the cyma and cavetto, being of weaker form, should only be used for the cover or shelter of the other parts. The torus and astragal, bearing a resemblance to a rope, appear calculated to bind and fortify the parts to which they are applied; while the use of the fillet and scotia is to separate one moulding from another, and to give a variety to the general appearance.

The ovolo and cyma are mostly placed in situations above the level of the eye: when placed below it, they should only be applied to crowning members. The place of the scotia is universally below the level of the eye. When the fillet is very wide, and used under the cyma of a cornice, it is called a corona; if under a corona, it is called a band.

The curved contours of mouldings are portions either of circles or ellipses: the Greeks always preferred the latter.



Fillet
Plate-band
Drip
Ovolo
A
C
D
Ovolo
C
A
Ovolo
B
B
Torus
A
Cavetto-reversed
C
B
Bead
A
Cavetto
C
Scotia
B
A
Cyma-recta
D
A
B
C
A
D
Cyma-reversa
C
B
Scotia
A
B
Cyma-recta
D
A
B
C
Ogee or Talon
C
B
A
D
Ogee
B
C
D
A
Ogee
P
B
L
C
A
I
N
Ogee
P
B
L
C
A
I
N

SECTION II.

OF THE ORDERS OF ARCHITECTURE, AND THEIR PRINCIPAL PARTS.

1. *How many orders of architecture are there?*

Five: the Tuscan, the Doric, the Ionic, the Corinthian, and the Composite.

2. *How many parts do we distinguish in each of the five orders?*

Three: the pedestal, the column, and the entablature.

3. *Of how many parts is the pedestal composed?*

Three: the plinth, the die, and the cornice.

4. *Of how many parts is the column composed?*

Three: the base, the shaft, and the capital.

5. *Of how many parts is the entablature composed?*

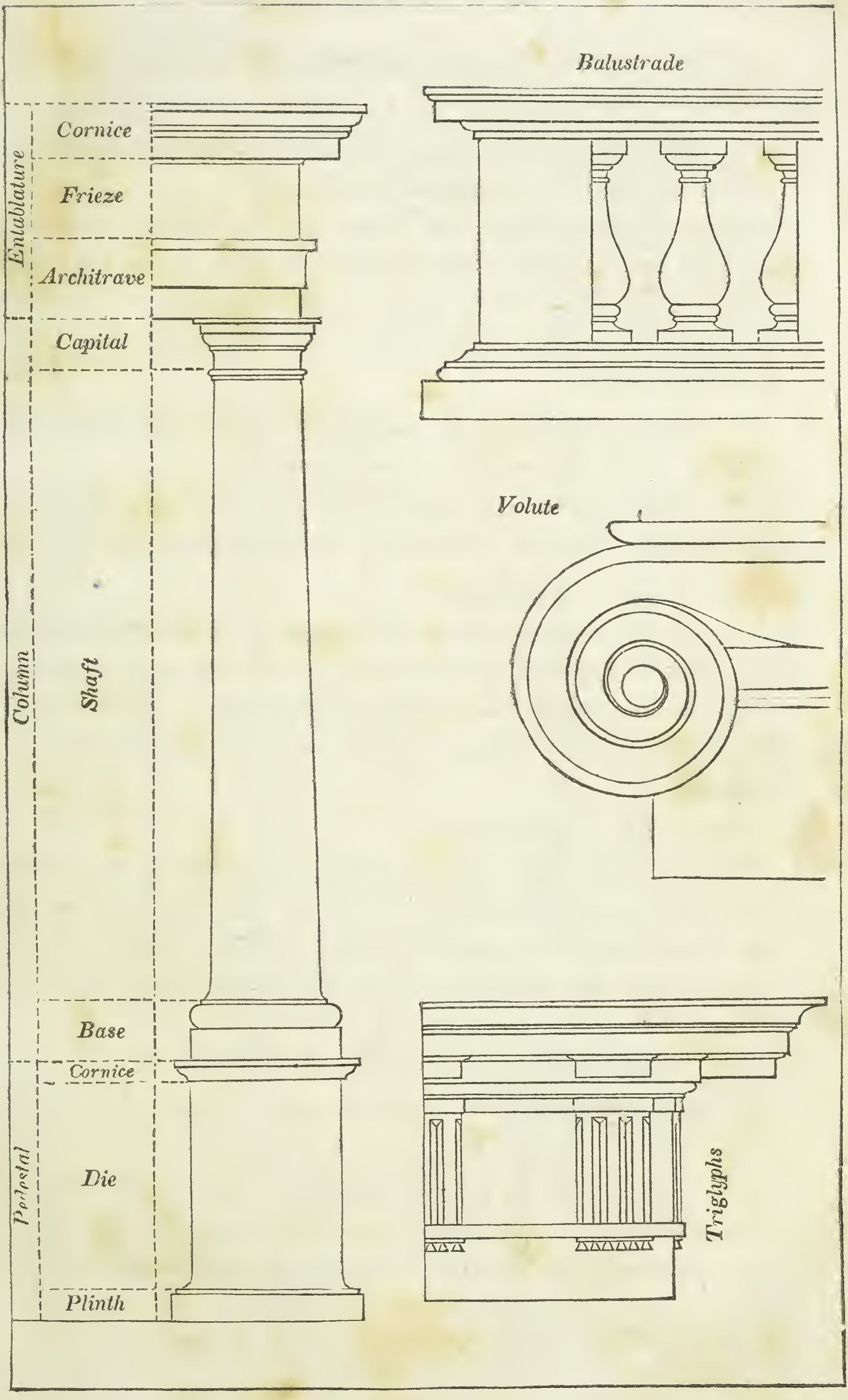
Three: the architrave, the frieze, and the cornice.

6. *Are these three principal parts always found in each of the orders?*

Not always; for, in giving the name of an order to an edifice, regard is not always had to the columns, but sometimes to the proportions observed in its construction. Sometimes, even, there are no columns; and often the pedestal is replaced by the plinth only.

7. *How are the five orders distinguished?*

The Tuscan is distinguished by the simplicity of its members, having no ornament; the Doric by the triglyphs which ornament its frieze; the Ionic by the volutes of its capital; the Corinthian by the leaves which ornament its capital;



Entablature
Cornice
Frieze
Architrave
Capital
Column
Shaft
Base
Cornice
Pedestal
Die
Plinth
Balustrade
Volute
Triglyphs

and the Composite by the Corinthian capital, united with the volutes of the Ionic.

8. *What proportion exists between the diameter and height of the columns in the different orders?*

In the Tuscan order, the height of the column, including its base and capital, is seven times the diameter of the shaft at the base; that of the Doric column eight times; that of the Ionic nine times; and that of the Corinthian and Composite ten times.

9. *What proportions are established between the three principal parts, in the orders of architecture?*

In all the orders, the pedestal is one third the height of the column, and the entablature is one quarter the height.

10. *What is a module?*

In all the orders except the Doric, it is the diameter of the shaft at the base: in the Doric, it is the semi-diameter. This is according to Gwilt's Architecture, published at London in 1839. Some authors call the semi-diameter a module.

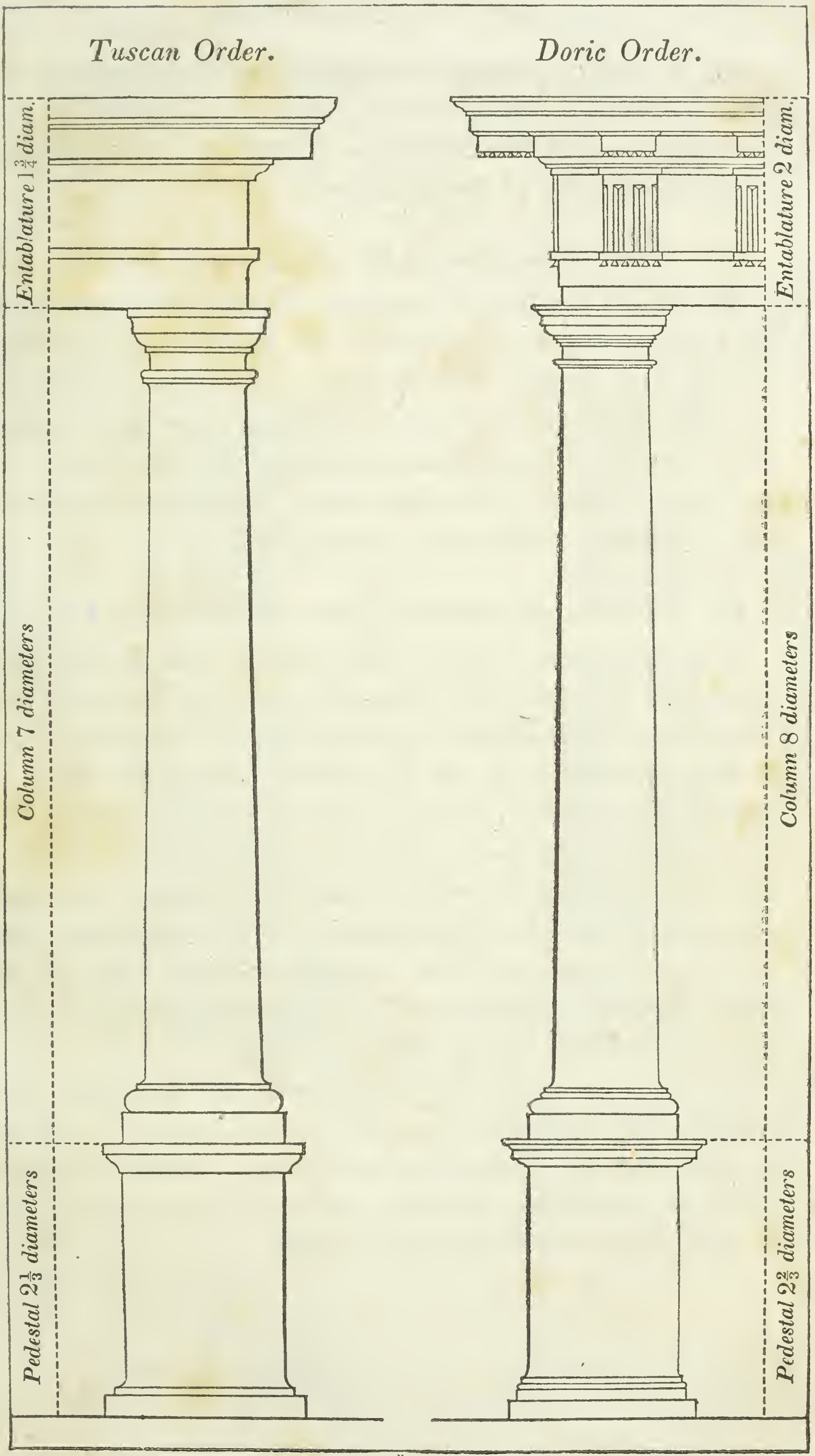
11. *What is a minute?*

In the Doric order, the module is supposed to be divided into thirty equal parts, and in each of the other orders into sixty, and each of the equal parts is called a minute: hence a minute is one-sixtieth part of the diameter of the shaft at its base.

12. *If the diameter of a shaft is two feet at the base, what will be the height of the structure in each of the five orders?*

FOR THE TUSCAN ORDER.

	$2 \times 7 =$	14 feet	= height of column, (Art. 8)
add one-third	=	4ft. 8in.	= height of pedestal, (Art. 9)
add one-fourth	=	3ft. 6in.	= height of entablature, (Art. 9.)
Total height	=	22ft. 2in.	


Tuscan Order.
Doric Order.
Entablature $1\frac{3}{4}$ diam.
Column 7 diameters
Pedestal $2\frac{1}{3}$ diameters
Entablature 2 diam.
Column 8 diameters
Pedestal $2\frac{2}{3}$ diameters

By a similar process, we should find the height of the Doric to be 25 feet 4 inches; that of the Ionic, 28 feet 6 inches; that of the Corinthian, 31 feet 8 inches; and that of the Composite, 31 feet 8 inches.

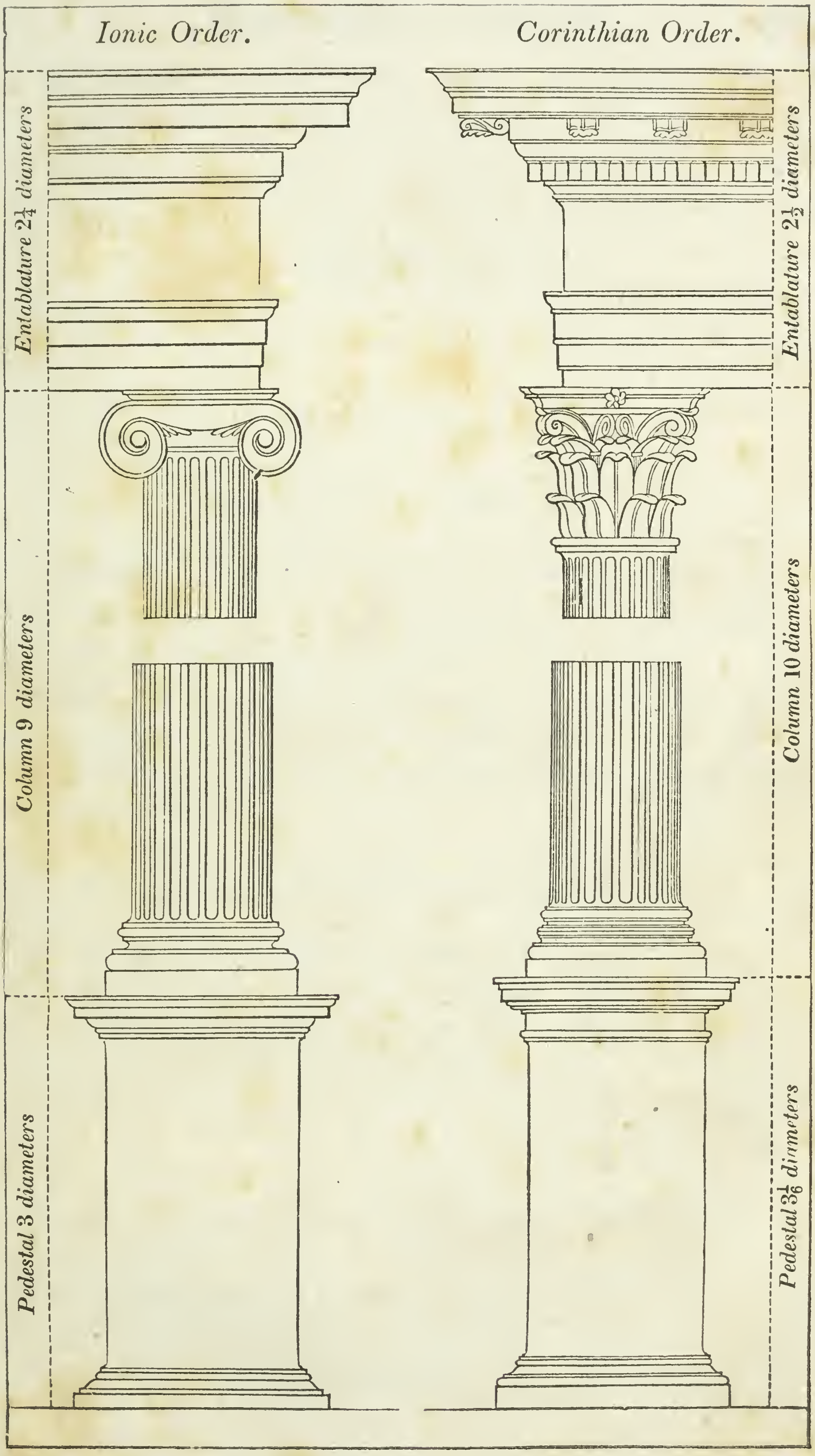
13. *What is the form of the shafts of the columns?*

The shafts diminish in diameter as they rise: sometimes the tapering begins at the foot of the shaft; sometimes from a point one quarter from the base, and sometimes from a point one-third; and in some examples there is a swelling in the middle. The difference between the diameter at top and that at bottom, is seldom more than one-sixth of the least diameter, or less than one-eighth.

14. *What do you remark of the entablatures?*

The entablature and its subdivisions, though architects frequently vary from the proportions, may as a general rule be set down as exhibited in the drawings. The total height of the entablature, in all the orders except the Doric, is divided into 10 parts, three of which are given to the architrave, three to the frieze, and four to the cornice. But in the Doric order, the whole should be divided into eight parts, two given to the architrave, three to the frieze, and three to the cornice. The mouldings, which form the detail of these leading features, are best learned by reference to representations of the orders at large.

In the Ionic order the entablatures are generally very simple The architrave has one or two fasciæ; the frieze is plain, and the cornice has four parts. In the Composite order, the entablature is large for so slender an order; yet it is on many accounts very beautiful.


Ionic Order.
Corinthian Order.
Entablature $2\frac{1}{4}$ diameters
Column 9 diameters
Pedestal 3 diameters
Entablature $2\frac{1}{2}$ diameters
Column 10 diameters
Pedestal $3\frac{1}{6}$ diameters

BOOK V.

SECTION I.

MENSURATION OF SURFACES.

1. *What do you understand by the unit of length?*

If the length of a line be computed in feet, one foot is the unit of the line, and is called the *linear unit.*

If the length of a line be computed in yards, one yard is the linear unit. If it be computed in rods, one rod is the linear unit; and if it be computed in chains, one chain is the linear unit.

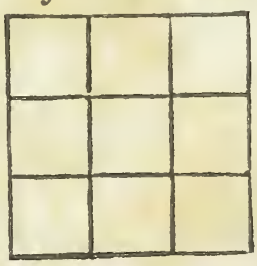
2. *What do you understand by the unit of surface?*

If we describe a square on the unit of length, such square is called the *unit of surface.* Thus, if the linear unit be 1 foot, one square foot will be the unit of surface.

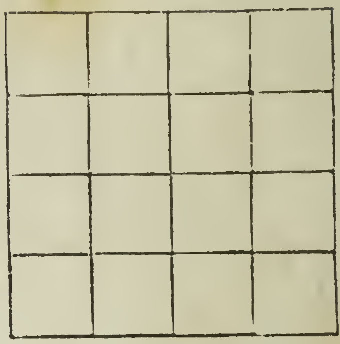
3. *How many square feet are there in a square yard?*

If the linear unit is 1 yard, one square yard will be the unit of surface; and this square yard contains 9 square feet.

1 yard = 3 feet.



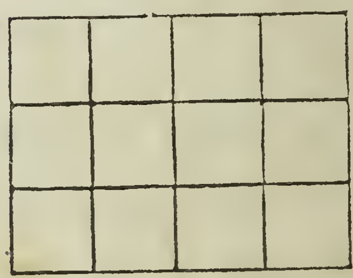
1 chain = 4 rods.



4. *How many square rods are there in a square chain?*

If the linear unit is 1 chain, the unit of surface will be 1 square chain, which will contain 16 square rods.

5. *How do you find the number of square feet contained in a rectangle?*



If we have a rectangle whose base is 4 feet, and altitude 3 feet, it is evident that it will contain 12 square feet. These 12 square feet are the *measure of the surface* of the rectangle.

6. *How do you find the number of squares contained in any rectangle?*

It is plain that the number of squares in any rectangle, will be expressed by the units of its base, multiplied by the units in its altitude. This product is called the measure of the rectangle

7. *What do you mean by the rectangle of two lines?*

In geometry, we often say, the rectangle of two lines, by which we mean, the rectangle of which those lines are the two adjacent sides.

8. *What is the area of a figure?*

The measure of its surface.

9. *What is the unit of the number which expresses the area?*

It is a square, of which the linear unit is the side.

10. *How do you find the area of a rectangle?*

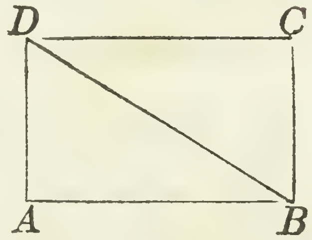
The area of a rectangle is equal to the product of its base by its altitude. If the base of a rectangle is 30 yards, and the altitude 5 yards, the area will be 150 square yards.

11. *What is the area of a square equal to?*

The area of a square is equal to the product of its two equal sides; that is, to the square of one of its sides.

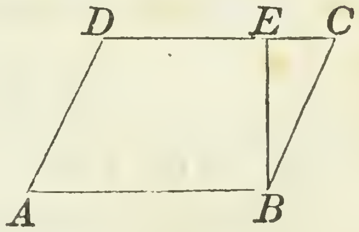
12. *How does the diagonal of a rectangle divide it?*

The diagonal DB divides the rectangle $ABCD$ into two equal triangles. Hence, a triangle is half a rectangle, having the same base and altitude.



13. *What is the altitude of a parallelogram?*

The altitude of a parallelogram is the perpendicular distance between two of its parallel sides. Thus, EB is the altitude of the parallelogram $ABCD$.

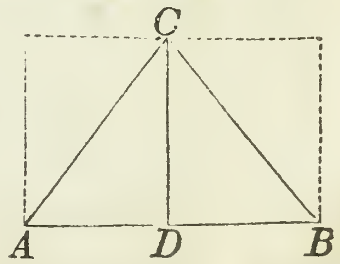


14. *What part of a parallelogram is a triangle, having the same base and the same altitude?*

A triangle is also half a parallelogram, having the same base and altitude.

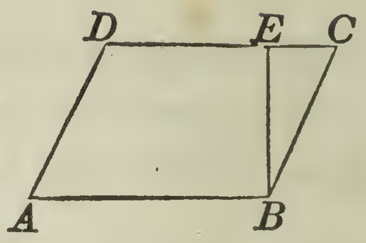
15. *What is the area of a triangle equal to?*

The area of a triangle is equal to half the product of the base by the altitude; for, the base multiplied by the altitude gives a rectangle which is double the triangle. Thus, the area of the triangle ABC is equal to half the product of $AB \times CD$.



If the base of a triangle is 12, and the altitude 8 yards, the area will be 48 square yards.

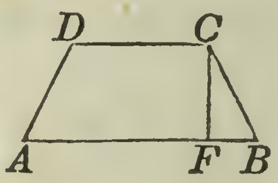
16. *What is the area of a parallelogram?*



The area of a parallelogram is equal to its base multiplied by its altitude. Thus, the area of the parallelogram *ABCD* is equal to $AB \times BE$.

If the base is 20, and altitude 15 feet, the area will be 300 square feet.

17. *What is the area of a trapezoid?*



The area of a trapezoid is equal to half the sum of its parallel sides multiplied by the perpendicular distance between them. Thus,

$$\text{area } ABCD = \tfrac{1}{2}(AB + CD) \times CF.$$

18. *With what is land generally measured?*

Surveyors, in measuring land, generally use a chain, called Gunter's chain. This chain is four rods, or 66 feet in length, and is divided into 100 links.

19. *What is an acre?*

An *acre* is a surface equal in extent to 10 square chains; that is, equal to a rectangle of which one side is ten chains, and the other side one chain.

20. *What is a quarter of an acre called?*

One quarter of an acre is called a *rood.*

21. *How many square rods in an acre?*

Since the chain is 4 rods in length, 1 square chain contains 16 square rods; and therefore, an acre, which is 10 square chains, contains 160 square rods, and a rood contains 40 square rods. The square rods are called perches.

22. *How is land generally computed?*

Land is generally computed in acres, roods, and perches, which are respectively designated by the letters *A. R. P.*

23. *If the linear dimensions are chains or links, how do you find the acres?*

When the linear dimensions of a survey are chains or links, the area will be expressed in square chains or square links, and it is necessary to form a rule for reducing this area to acres, roods, and perches. For this purpose, let us form the following

TABLE.

1 square chain = 10000 square links.
1 acre = 10 square chains = 100000 square links.
1 acre = 4 roods = 160 perches.
1 square mile = 6400 square chains = 640 acres.

24. *If the linear dimensions are links, how do you find the acres?*

When the linear dimensions are links, the area will be expressed in square links, and may be reduced to acres by dividing by 100000, the number of square links in an acre: that is, by pointing off five decimal places from the right hand.

If the decimal part be then multiplied by 4, and five places of decimals pointed off from the right hand, the figures to the left will express the roods.

If the decimal part of this result be now multiplied by 40, and five places for decimals pointed off, as before, the figures to the left will express the perches.

If one of the dimensions be in links, and the other in chains, the chains may be reduced to links by annexing two ciphers: or, the multiplication may be made without annexing the ciphers, and the product reduced to acres and

decimals of an acre, by pointing off three decimal places at the right hand.

When both the dimensions are in chains, the product is reduced to acres by dividing by 10, or pointing off one decimal place.

From which we conclude that,

1st. If links be multiplied by links, the product is reduced to acres by pointing off five decimal places from the right hand.

2d. If chains be multiplied by links, the product is reduced to acres by pointing off three decimal places from the right hand.

3d. If chains be multiplied by chains, the product is reduced to acres by pointing off one decimal place from the right hand.

25. *How do you find the number of square feet in an acre?*

Since there are 16.5 feet in a rod, a square rod is equal to

$$16.5 \times 16.5 = 272.25 \text{ square feet.}$$

If the last number be multiplied by 160, the number of square rods in an acre, we shall have

$$272.25 \times 160 = 43560 = \text{the square-feet in an acre.}$$

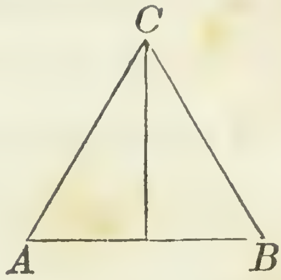
OF THE TRIANGLE.

26. *How do you find the area of a triangle, when the base and altitude are known?*

1st. Multiply the base by the altitude, and half the product will be the area.

Or, 2d. Multiply the base by half the altitude, and the product will be the area.

EXAMPLES.



1. Required the area of the triangle ABC, whose base AB is 10.75 feet and altitude 7.25 feet.

Operation.

We first multiply the base by the altitude, and then divide the product by 2.

$10.75 \times 7.25 = 77.9375$
and
$77.9375 \div 2 = 38.96875$
$=$ area.

2. What is the area of a triangle whose base is 18 feet 4 inches, and altitude 11 feet 10 inches?

Ans. 108 *sq. ft* 5′ 8″.

3. What is the area of a triangle whose base is 12.25 chains, and altitude 8.5 chains?

Ans. 5 *A.* 0 *R.* 33 *P.*

4. What is the area of a triangle whose base is 20 feet, and altitude 10.25 feet?

Ans. 102.5 *sq. ft.*

5. Find the area of a triangle whose base is 625 and altitude 520 feet.

Ans. 162500 *sq. ft.*

6. Find the number of square yards in a triangle whose base is 40 and altitude 30 feet.

Ans. $66\frac{2}{3}$ *sq. yds.*

7. What is the area of a triangle whose base is 72.7 yards, and altitude 36.5 yards?

Ans. 1326.775 *sq. yds.*

8. What is the content of a triangular field whose base is 25.01 chains, and perpendicular 18.14 chains?

Ans. 22 *A.* 2 *R.* 29 *P.*

9. What is the content of a triangular field whose base is 15.48 chains, and altitude 9.67 chains?

Ans. 7 *A.* 1 *R.* 38 *P.*

27. *How do you find the area of a triangle when the three sides are given?*

1st. Add the three sides together and take half their sum.

2d. From this half sum take each side separately.

3d. Multiply together the half sum and each of the three remainders, and then extract the square root of the product, which will be the required area.

EXAMPLES.

1. Find the area of a triangle whose sides are 20, 30, and 40 rods.

20	45	45	45
30	20	30	40
40	25 1st rem.	15 2d rem.	5 3d rem.
2)90			
45 half sum.			

Then, to obtain the product, we have

$$45 \times 25 \times 15 \times 5 = 84375;$$

from which we find

$$\text{area} = \sqrt{84375} = 290.4737 \text{ perches.}$$

2. How many square yards of plastering are there in a triangle, whose sides are 30, 40, and 50 feet?

Ans. $66\frac{2}{3}$.

3. The sides of a triangular field are 49 chains, 50.25 chains, and 25.69 chains: what is its area?

Ans. 61 *A.* 1 *R.* 39.68 *P.*

4. What is the area of an isosceles triangle, whose base is 20, and each of the equal sides 15?

Ans. 111.803.

5. How many acres are there in a triangle whose three sides are 380, 420, and 765 yards?

Ans. 9 *A.* 0 *R.* 38 *P.*

6. How many square yards in a triangle whose sides are 13, 14, and 15 feet?

Ans. $9\frac{1}{3}$.

7. What is the area of an equilateral triangle whose side is 25 feet?

Ans. 270.6329 *sq. ft.*

8. What is the area of a triangle whose sides are 24, 36, and 48 yards?

Ans. 418.282 *sq. yds.*

28. *How do you find the hypothenuse of a right-angled triangle when the base and perpendicular are known?*

1st. Square each of the sides separately.

2d. Add the squares together.

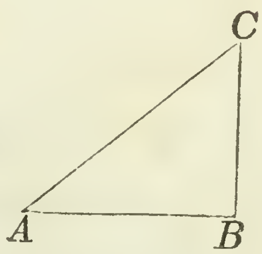
3d. Extract the square root of the sum, which will be the hypothenuse of the triangle.

EXAMPLES.

1. In the right-angled triangle ABC, we have

$$AB = 30 \text{ feet}, \quad BC = 40 \text{ feet}.$$

to find AC.



We first square each side, and then take the sum, of which we extract the square root, which gives

$$AC = \sqrt{2500} = 50 \text{ feet}.$$

Operation.

$$\overline{30}^2 = 900$$
$$\overline{40}^2 = 1600$$
$$\text{sum} = \underline{\overline{2500}}$$

2. The wall of a building, on the brink of a river, is 120 feet high, and the breadth of the river 70 yards: what is

the length of a line which would reach from the top of the wall to the opposite edge of the river?

Ans. 241.86 *ft.*

3. The side roofs of a house of which the eaves are of the same height, form a right angle at the top. Now, the length of the rafters on one side is 10 feet, and on the other 14 feet: what is the breadth of the house?

Ans. 17.204 *ft.*

4. What would be the width of the house, in the last example, if the rafters on each side were 10 feet?

Ans. 14.142 *ft.*

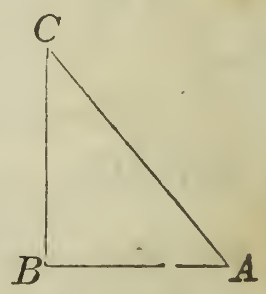
5. What would be the width, if the rafters on each side were 14 feet?

Ans. 19.7989 *ft.*

29. *If the hypothenuse and one side of a right-angled triangle are known, how do you find the other side?*

Square the hypothenuse and also the other given side, and take their difference: extract the square root of this difference, and the result will be the required side.

EXAMPLES.



1. In the right-angled triangle ABC, there are given

$AC = 50$ feet, and $AB = 40$ feet;

required the side BC.

We first square the hypothenuse and the other side, after which we take the difference, and then extract the square root, which gives

Operation.

$$\overline{50}^2 = 2500$$
$$\overline{40}^2 = 1600$$
$$\text{Diff.} = \underline{900}$$

$$BC = \sqrt{900} = 30 \text{ feet.}$$

2. The height of a precipice on the brink of a river is 103 feet, and a line of 320 feet in length will just reach from the top of it to the opposite bank: required the breadth of the river.

Ans. 302.9703 *ft.*

3. The hypothenuse of a triangle is 53 yards, and the perpendicular 45 yards: what is the base?

Ans. 28 *yds.*

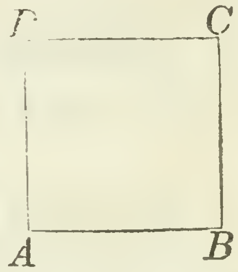
4. A ladder 60 feet in length, will reach to a window 40 feet from the ground on one side of the street, and by turning it over to the other side, it will reach a window 50 feet from the ground: required the breadth of the street.

Ans. 77.8875 *ft.*

AREA OF THE SQUARE.

30. *How do you find the area of a square, a rectangle, or a parallelogram?*

Multiply the base by the perpendicular height, and the product will be the area.



1. Required the area of the square $ABCD$, each of whose sides is 36 feet.

We multiply two sides of the square together, and the product is the area in square feet.

Operation.

$36 \times 36 = 1296$ *sq. ft.*

2. How many acres, roods, and perches, in a square whose side is 35.25 chains?

Ans. 124 *A.* 1 *R.* 1 *P.*

3. What is the area of a square whose side is 8 feet 4 inches? (See Arithmetic, § 171.)

Ans. 69 *ft.* 5′ 4″.

4. What is the content of a square field whose side is 46 rods?

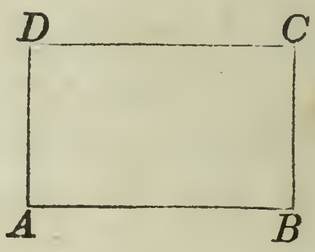
Ans. 13 *A.* 0 *R.* 36 *P.*

5. What is the area of a square whose side is 4769 yards?

Ans. 22743361 *sq. yds.*

AREA OF THE RECTANGLE.

1. To find the area of a rectangle *ABCD*, of which the base *AB* = 45 yards, and the altitude *AD* = 15 yards.



Here we simply multiply the base by the altitude, and the product is the area.

Operation.

45 × 15 = 675 *sq. yds.*

2. What is the area of a rectangle whose base is 14 feet 6 inches, and breadth 4 feet 9 inches?

Ans. 68 *sq. ft.* 10′ 6″.

3. Find the area of a rectangular board whose length is 112 feet, and breadth 9 inches.

Ans. 84 *sq. ft.*

4. Required the area of a rectangle whose base is 10.51, and breadth 4.28 chains.

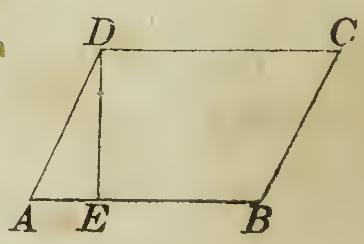
Ans. 4 *A.* 1 *R.* 39.7 *P* +.

5. Required the area of a rectangle whose base is 12 feet 6 inches, and altitude 9 feet 3 inches.

Ans. 115 *sq. ft.* 7′ 6″.

AREA OF THE PARALLELOGRAM.

1. What is the area of the parallelogram *ABCD*, of which the base *AB* is 64 feet, and altitude *DE*, 36 feet?



We multiply the base 64, by the perpendicular height 36, and the product is the required area.

Operation.

$$64 \times 36 = 2304 \ sq. \ ft.$$

2. What is the area of a parallelogram whose base is 12.25 yards, and altitude 8.5 yards?

Ans. 104.125 *sq. yds.*

3. What is the area of a parallelogram whose base is 8.75 chains, and altitude 6 chains?

Ans. 5 *A.* 1 *R.* 0 *P.*

4. What is the area of a parallelogram whose base is 7 feet 9 inches, and altitude 3 feet 6 inches?

Ans. 27 *sq. ft.* 1′ 6″.

5. What is the area of a parallelogram whose base is 10.50 chains, and breadth 14.28 chains?

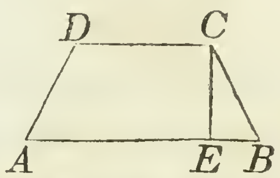
Ans. 14 *A.* 3 *R.* 30 *P* +.

AREA OF THE TRAPEZOID.

31. *How do you find the area of a trapezoid?*

Multiply the sum of the parallel sides by the perpendicular distance between them, and then divide the product by two: the quotient will be the area.

EXAMPLES.



1. Required the area of the trapezoid $ABCD$, having given

$AB = 321.51$ ft., $DC = 214.24$ ft., and $CE = 171.16$ ft.

We first find the sum of the sides, and then multiply it by the perpendicular height, after which, we divide the product by 2, for the area.

Operation.

$321.51 + 214.24 = 535.75 =$ sum of parallel sides.

Then,

$535.75 \times 171.16 = 91698.97$

and, $\dfrac{91698.97}{2} = 45849.485$

$=$ the area.

2. What is the area of a trapezoid, the parallel sides of which are 12.41 and 8.22 chains, and the perpendicular distance between them 5.15 chains?

Ans. 5 *A.* 1 *R.* 9.956 *P.*

3. Required the area of a trapezoid whose parallel sides are 25 feet 6 inches, and 18 feet 9 inches, and the perpendicular distance between them 10 feet and 5 inches.

Ans. 230 *sq. ft.* 5′ 7″.

4. Required the area of a trapezoid whose parallel sides are 20.5 and 12.25, and the perpendicular distance between them 10.75 yards.

Ans. 176.03125 *sq. yds.*

5. What is the area of a trapezoid whose parallel sides are 7.50 chains, and 12.25 chains, and the perpendicular height 15.40 chains?

Ans. 15 *A.* 0 *R.* 32.2 *P.*

6. What is the content when the parallel sides are 20 and 32 chains, and the perpendicular distance between them 26 chains?

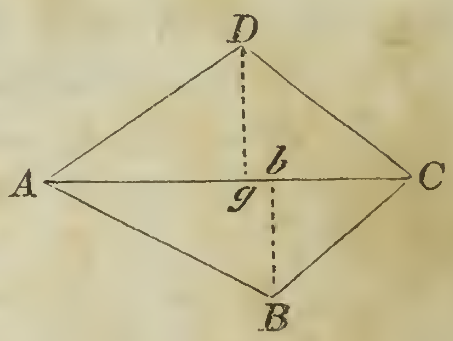
Ans. 67 *A.* 2 *R.* 16 *P.*

AREA OF A QUADRILATERAL.

32. *How do you find the area of a quadrilateral?*

Measure the four sides of a quadrilateral, and also one of the diagonals: the quadrilateral will thus be divided into two triangles, in both of which all the sides will be known. Then, find the areas of the triangles separately, and their sum will be the area of the quadrilateral.

EXAMPLES.



1. Suppose that we have measured the sides and diagonal AC, of the quadrilateral $ABCD$, and found

$$AB = 40.05 \text{ ch}, \quad CD = 29.87 \text{ ch},$$
$$BC = 26.27 \text{ ch}, \quad AD = 37{,}07 \text{ ch},$$
and $$AC = 55 \text{ ch}:$$
required the area of the quadrilateral.

Ans. 101 *A.* 1 *R.* 15 *P.*

REMARK.—Instead of measuring the four sides of the quadrilateral, we may let fall the perpendiculars Bb, Dg, on the diagonal AC. The area of the triangles may then be determined by measuring these perpendiculars and the diagonal AC. The perpendiculars are $Dg = 18.95$ ch, and $Bb = 17.92$ ch.

2. Required the area of a quadrilateral whose diagonal is 80.5 and two perpendiculars 24.5 and 30.1 feet.

Ans. 2197.65 *sq. ft.*

3. What is the area of a quadrilateral whose diagonal is 108 feet 6 inches, and the perpendiculars 56 feet 3 inches, and 60 feet 9 inches?

Ans. 6347 *sq. ft.* 3′.

4. How many square yards of paving in a quadrilateral whose diagonal is 65 feet, and the two perpendiculars 28 and $33\frac{1}{2}$ feet?

Ans. $222\frac{1}{12}$ *sq. yds.*

5. Required the area of a quadrilateral whose diagonal is 42 feet, and the two perpendiculars 18 and 16 feet.

Ans. 714 *sq. ft.*

6. What is the area of a quadrilateral in which the diagonal is 320.75 chains, and the two perpendiculars 69.73 chains, and 130.27 chains?

Ans. 3207 *A.* 2 *R.*

33. *How do you find the area of a long and irregular figure, bounded on one side by a straight line?*

1st. Divide the right line or base into any number of

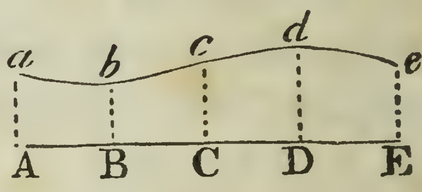
equal parts, and measure the breadth of the figure at the points of division, and also at the extremities of the base.

2d. Add together the intermediate breadths, and half the sum of the extreme ones.

3d. Multiply this sum by the base line, and divide the product by the number of equal parts of the base.

EXAMPLES.

1. The breadths of an irregular figure, at five equidistant places, *A*, *B*, *C*, *D*, and *E*, being 8.20 ch, 7.40 ch, 9.20 ch, 10.20 ch, and 8.60 chains, and the whole length 40 chains; required the area.



$$\begin{array}{r l}
8.20 & \\
8.60 & \\
\hline
2)16.80 & \\
\hline
8.40 & \text{mean of the extremes.} \\
7.40 & \\
9.20 & \\
10.20 & \\
\hline
35.20 & \text{sum.} \\
\hline
\end{array}
\qquad
\begin{array}{r l}
35.20 & \\
40 & \\
\hline
4)1408.00 & \\
\hline
352.00 & \text{square ch.} \\
\hline
\end{array}$$

Ans. 35 *A.* 32 *P.*

2. The length of an irregular piece of land being 21 ch, and the breadths, at six equidistant points, being 4.35 ch, 5.15 ch, 3.55 ch, 4.12 ch, 5.02 ch, and 6.10 chains: required the area. *Ans.* 9 *A.* 2 *R.* 30 *P.*

3. The length of an irregular figure is 84 yards, and the breadths at six equidistant places are 17.4; 20.6; 14.2; 16.5; 20.1, and 24.4: what is the area?

Ans. 1550.64 *sq. yds.*

4. The length of an irregular field is 39 rods, and its breadths at five equidistant places are 4.8; 5.2; 4.1; 7.3, and 7.2 rods: what is its area?

Ans. 220.35 *sq. rods.*

5. The length of an irregular field is 50 yards, and its breadths at seven equidistant points are 5.5; 6.2; 7.3; 6; 7.5; 7; and 8.8 yards: what is its area?

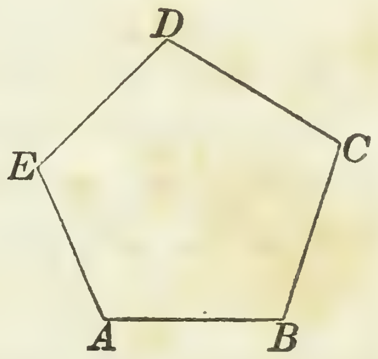
Ans. 342.916 *sq. yds.*

6. The length of an irregular figure being 37.6, and the breadths at nine equidistant places, 0; 4.4; 6.5; 7.6; 5.4; 8; 5.2; 6.5; and 6.1: what is the area?

Ans. 219.255.

OF POLYGONS.

34. *What is a regular Polygon?*



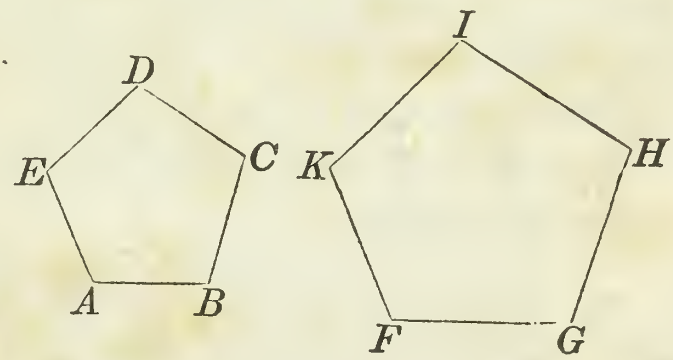
A regular polygon is one which has all its sides equal to each other, each to each, and all its angles equal to each other, each to each.

Thus, if the polygon $ABCDE$ be regular, we have

$$AB = BC = CD = DE = EA: \text{ also}$$
$$\text{angle } A = B = C = D = E.$$

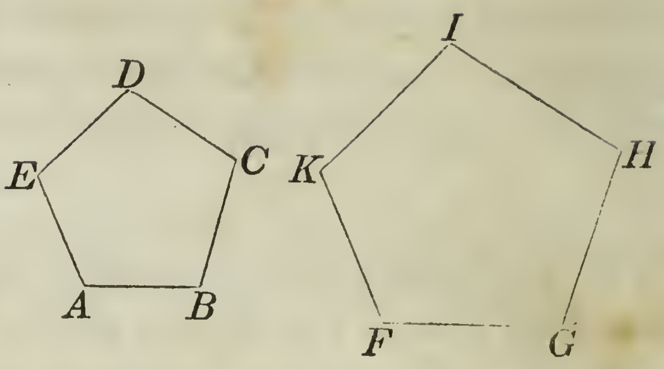
35. *What are similar polygons?*

Similar polygons are those which have the angles of the one equal to the angles of the other, each to each, and the sides about the equal angles proportional. Hence, similar polygons are alike in shape, but may differ in size.



The sides which are like situated in two similar polygons, are called *homologous sides*, and these sides are proportional to each other.

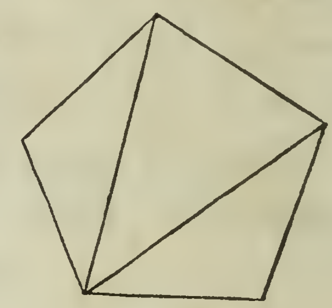
Thus, if $ABCDE$ and $FGHIK$ are two similar polygons: then angle $A = F$, $B = G$, $C = H$, $D = I$, and $E = K$.



Also, $AB : FG :: BC : GH$
and $AB : FG :: CD : HI$;
also, $AB : FG :: DE : IK$
and $AB : FG :: EA : KF$.

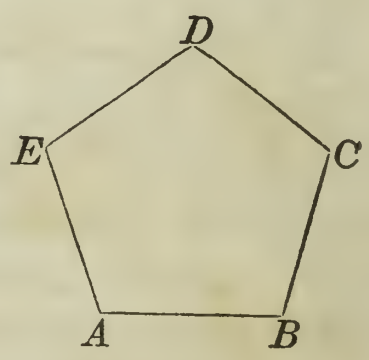
36. *Into how many triangles may any polygon be divided?*

Any polygon may be divided by diagonals, into as many triangles less two, as the polygon has sides. Thus, if the polygon has five sides, there will be three triangles; if it has six sides, there will be four; if seven sides, five; if eight sides, six; &c.



37. *What is the sum of all the inward angles of a polygon equal to?*

The sum of all the inward angles of any polygon is equal to twice as many right angles, wanting four, as the figure has sides. Thus, if the polygon has five sides, we have



$A + B + C + D + E = 10$ right angles $-$ 4 right angles $= 6$ right angles.

38. *What is the sum of the angles of a quadrilateral equal to?*

If the polygon is a quadrilateral, then the sum of the angles will be equal to four right angles.

39. *How do you find one of the angles of a regular polygon?*

When the polygon is regular, its angles will be equal to each other. If, then, the sum of the inward angles be divided by the number of angles, the quotient will be the value of one of the angles. We shall find the value in degrees, by simply placing 90° for the right angle.

40. *How do you find one of the angles of an equilateral triangle?*

The sum of all the angles of an equilateral triangle is equal to

$$6 \times 90^\circ - 4 \times 90^\circ = 540^\circ - 360^\circ = 180^\circ$$

and for each angle

$$180^\circ \div 3 = 60^\circ:$$

Hence, each angle of an equilateral triangle is equal to 60 degrees.

41. *How do you find one of the angles of a square or rectangle?*

The sum of all the angles of a square or rectangle is

$$8 \times 90^\circ - 4 \times 90^\circ = 720^\circ - 360^\circ = 360^\circ:$$

and for each of the angles

$$360^\circ \div 4 = 90^\circ.$$

42. *How do you find one of the angles of a regular pentagon?*

The sum of all the angles of a regular pentagon is equal to

$$10 \times 90^\circ - 4 \times 90^\circ = 900^\circ - 360^\circ = 540^\circ:$$

and for each angle

$$540^\circ \div 5 = 108^\circ.$$

43. *How do you find one of the angles of a regular hexagon?*

The sum of all the angles of a regular hexagon is equal to

$$12 \times 90^\circ - 4 \times 90^\circ = 1080^\circ - 360^\circ = 720^\circ:$$

and for each angle

$$720^\circ \div 6 = 120^\circ.$$

44. *How do you find one of the angles of a regular heptagon?*

The sum of the angles of a regular heptagon is equal to

$$14 \times 90^\circ - 4 \times 90^\circ = 1260^\circ - 360^\circ = 900^\circ:$$

and for one of the angles

$$900^\circ \div 7 = 128^\circ\ 34' +.$$

45. *How do you find one of the angles of a regular octagon?*

The sum of the angles of a regular octagon is equal to

$$16 \times 90^\circ - 4 \times 90^\circ = 1440^\circ - 360^\circ = 1080^\circ:$$

and for each angle

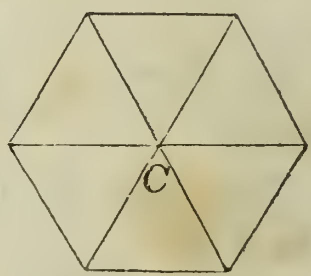
$$1080^\circ \div 8 = 135^\circ.$$

46. *How many figures can be arranged about a point so as to fill up the entire space?*

There are but three; the equilateral triangle, the square or rectangle, and the hexagon.

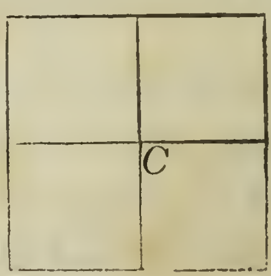
First.—Six equilateral triangles placed about the point C, will fill the entire space. For, each angle is equal to 60°, and their sum to

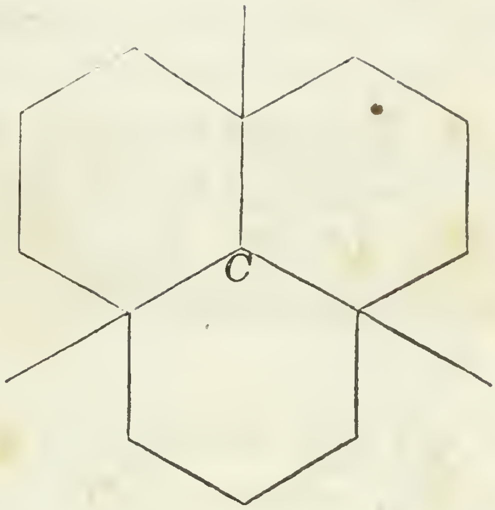
$$60^\circ \times 6 = 360^\circ.$$



Second.—Four squares, or rectangles, placed about C, will fill the entire space. For, each angle is equal to 90°, and their sum to

$$90^\circ \times 4 = 360^\circ.$$





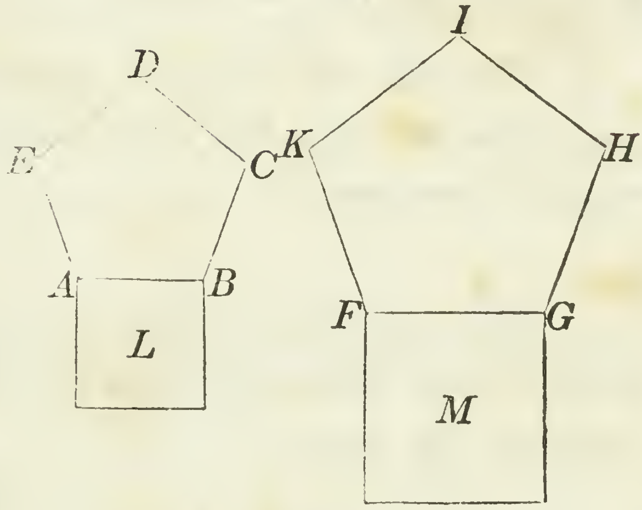
Third.—Three hexagons placed about C, will fill up the entire space. For, each angle is equal to 120°, and the sum of the three to

$$120° \times 3 = 360°.$$

47. *How are similar polygons to each other?*

Similar polygons are to each other as the squares described **on** their homologous sides.

Thus, the two similar polygons $ABCDE$, $FGHIK$, are to each other as the squares described on the homologous sides AB and FG: that is



$$ABCDE : FGHIK :: \text{square } L : \text{square } M.$$

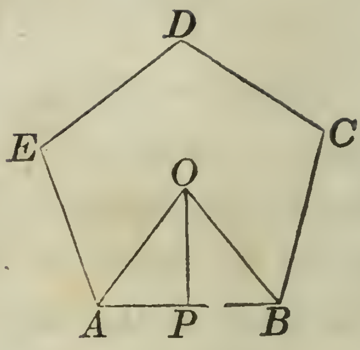
If AB were 4, the area $ABCDE$ would be 27.5276384. Now, if FG were 8, what would be the area $FGHIK$?

$$4^2 : 8^2 :: 27.5276384 : 110.1105536.$$

48. *How do you find the area of a regular polygon?*

Multiply half the perimeter of the figure by the perpendicular let fall from the centre on one of the sides, and the product will be the area.

EXAMPLES.



1. Required the area of the regular pentagon $ABCDE$, each of whose sides AB, BC, &c., is 25 feet, and the perpendicular OP, 17.2 feet.

We first multiply one side by the number of sides and divide the product by 2: this gives half the perimeter, which we multiply by the perpendicular for the area.

Operation.

$\frac{25 \times 5}{2} = 62.5 =$ half the perimeter. Then,

$62.5 \times 17.2 = 1075$ *sq. ft.* = the area.

2. The side of a regular pentagon is 20 yards, and the perpendicular from the centre on one of the sides 13.76382: required the area.

Ans. 688.191 *sq. yds.*

3. The side of a regular hexagon is 14, and the perpendicular from the centre on one of the sides 12.1243556: required the area.

Ans. 509.2229352 *sq. ft.*

4. Required the area of a regular hexagon whose side is 14.6, and perpendicular from the centre 12.64 feet.

Ans. 553.632 *sq. ft.*

5. Required the area of a heptagon whose side is 19.38, and perpendicular 20 feet.

Ans. 1356.6 *sq. ft.*

6. Required the area of an octagon whose side is 9.941 yards and perpendicular 12 yards.

Ans. 477.168 *sq. yds.*

49. The following table shows the areas of the ten regu-

lar polygons when the side of each is equal to 1. It also shows the length of the radius of the inscribed circle.

Number of sides.	Names.	Areas.	Radius of inscribed circle.
3	Triangle,	0.4330127	0.2886751
4	Square,	1.0000000	0.5000000
5	Pentagon,	1.7204774	0.6881910
6	Hexagon,	2.5980762	0.8660254
7	Heptagon,	3.6339124	1.0382617
8	Octagon,	4.8284271	1.2071068
9	Nonagon,	6.1818242	1.3737387
10	Decagon,	7.6942088	1.5388418
11	Undecagon,	9.3656404	1.2028437
12	Duodecagon,	11.1961524	1.8660254

50. *How do you find the area of any polygon from the above table?*

Since the areas of similar polygons are to each other as the squares described on their homologous sides, we have

$$1^2 : \text{tabular area} : : \text{any side squared} : \text{area}.$$

Hence, to find the area of a regular polygon.

1st. Square the side of the polygon.

2d. Multiply the square so found, by the tabular area set opposite the polygon of the same number of sides, and the product will be the required area.

EXAMPLES.

1. What is the area of a regular hexagon whose side is 20?

$$\overline{20}^2 = 400 \quad \text{and tabular area} = 2.5980762.$$

Hence,

$$2.5980762 \times 400 = 1039.23048 = \text{the area}.$$

2. What is the area of a pentagon whose side is 25?

Ans. 1075.298375.

3. What is the area of a heptagon whose side is 30?
Ans. 3270.52116.

4. What is the area of an octagon whose side is 10 feet?
Ans. 482.84271 *sq. ft.*

5. The side of a nonagon is 50: what is its area?
Ans. 15454.5605.

6. The side of an undecagon is 20: what is its area?
Ans. 3746.25616.

7. The side of a duodecagon is 40: what is its area?
Ans. 17913.84384.

8. Required the area of an octagon whose side is 16.
Ans. 1236.0773.

9. Required the area of a decagon whose side is 20.5.
Ans. 3233.4912.

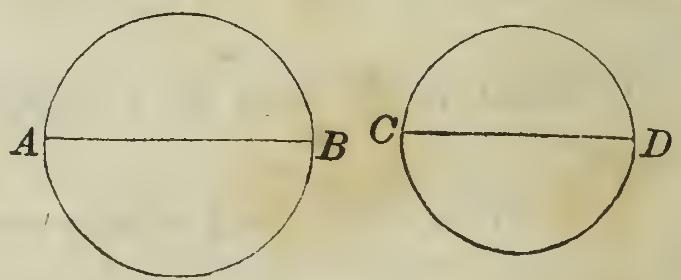
10. Required the area of a nonagon whose side is 36.
Ans. 8011.6442.

11. Required the area of a duodecagon whose side is 125.
Ans. 174939.881.

OF THE CIRCLE.

51. *How are the circumferences of circles to each other?*

The circumferences of circles are proportional to their diameters. If we represent the diameter AB by D, and the circumference



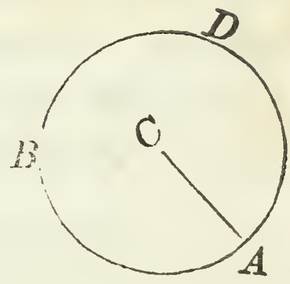
of the circle by C, and the diameter CD by d, and the circumference by c, we shall have

$$D : d :: C : c.$$

52. *How many times greater is the circumference than the diameter of a aircle?*

The circumference of a circle is a little more than three times greater than the diameter. If the diameter is 1, the circumference will be 3.1416.

53. *What is the area of a circle equal to?*



The area of a circle is equal to the product of half the radius, into the circumference. Thus, the area of the circle whose centre is C, is equal to half the radius CA, multiplied by the circumference: that is,

$$\text{area} = \tfrac{1}{2}CA \times \text{circumference } ABD.$$

54. *How do you find the circumference of a circle when the diameter is known?*

Multiply the diameter by 3.1416, and the product will be the circumference.

EXAMPLES.

1. What is the circumference of a circle whose diameter is 17?

We simply multiply the number 3.1416 by the diameter, and the product is the circumference.

Operation.

$3.1416 \times 17 = 53.4072$

which is the circumference.

2. What is the circumference of a circle whose diameter is 40 feet?

Ans. 125.664 *ft.*

3. What is the circumference of a circle whose diameter is 12 feet?

Ans. 37.6992 *ft.*

4. What is the circumference of a circle whose diameter is 22 yards?

Ans. 69.1152 *yds.*

5. What is the circumference of the earth—the mean diameter being about 7921 miles?

Ans. 24884.6136 *miles.*

55. *How do you find the diameter of a circle when the circumference is known?*

Divide the circumference by the number 3.1416, and the quotient will be the diameter.

EXAMPLES.

1. The circumference of a circle is 69.1152 yards: what is the diameter?

We simply divide the circumference by 3.1416, and the quotient 22 is the diameter sought.

Operation.

```
3.1416)69.1152(22
        62832
        -----
         62832
         62832
         -----
```

2. What is the diameter of a circle whose circumference is 11652.1944 feet?

Ans. 3709 *ft.*

3. What is the diameter of a circle whose circumference is 6850?

Ans. 2180.4176.

4. What is the diameter of a circle whose circumference is 50?

Ans. 15.915.

5. If the circumference of a circle is 25000.8528, what is the diameter?

Ans. 7958.

56. *How do you find the length of a circular arc, when the number of degrees which it contains, and the radius of the circle are known?*

Multiply the number of degrees by the decimal .01745, and the product arising by the radius of the circle.

EXAMPLES.

1. What is the length of an arc of 30 degrees, in a circle whose radius is 9 feet?

We merely multiply the given decimal by the number of degrees, and by the radius.

Operation.

$.01745 \times 30 \times 9 = 4.7115$, which is the length of the required arc.

Remark.—When the arc contains degrees and minutes, reduce the minutes to the decimals of a degree, which is done by dividing them by 60.

2. What is the length of an arc containing $12^\circ\ 10'$ or $12\frac{1}{6}^\circ$, the diameter of the circle being 20 yards?

Ans. 2.1231.

3. What is the length of an arc of $10^\circ\ 15'$ or $10\frac{1}{4}^\circ$, in a circle whose diameter is 68?

Ans. 6.0813.

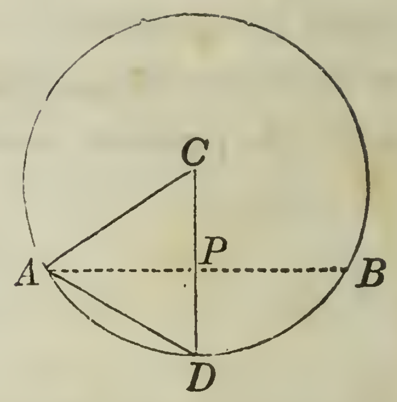
57. *How do you find the length of the arc of a circle when the chord and radius are given?*

1st. Find the chord of half the arc.

2d. From eight times the chord of half the arc, subtract the chord of the whole arc, and divide the remainder by three, and the quotient will be the length of the arc, nearly.

EXAMPLES.

1. The chord $AB = 30$ feet, and the radius $AC = 20$ feet: what is the length of the arc ADB?



First, draw CD perpendicular to the chord AB: it will bisect the chord at P, and the arc of the chord at D. Then $AP = 15$ feet. Hence

$$\overline{AC}^2 - \overline{AP}^2 = \overline{CP}^2: \text{ that is}$$

$$400 - 225 = 175, \text{ and } \sqrt{175} = 13.228 = CP.$$

Then, $CD - CP = 20 - 13.228 = 6.772 = DP.$

Again, $AD = \sqrt{\overline{AP}^2 + \overline{PD}^2} = \sqrt{225 + 45.859984}:$

hence, $AD = 16.4578 =$ chord of the half arc.

Then, $\dfrac{16.4578 \times 8 - 30}{3} = 33.8874 =$ arc ADB.

2. What is the length of an arc, the chord of which is 24 feet, and the radius of the circle 20 feet?

Ans. 25.7309 *ft.*

3. The chord of an arc is 16, and the diameter of the circle 20: what is the length of the arc?

Ans. 18.5178.

4. The chord of an arc is 50, and the chord of half the arc is 27: what is the length of the arc?

Ans. $55\frac{1}{3}$.

58. *How do you find the area of a circle when the diameter and circumference are both known?*

Multiply the circumference by half the radius, and the product will be the area.

EXAMPLES.

1. What is the area of a circle whose diameter is 10, and circumference 31.416?

If the diameter be 10, the radius is 5, and half the radius $2\frac{1}{2}$: hence the circumference multiplied by $2\frac{1}{2}$ gives the area.

Operation.

$31.416 \times 2\frac{1}{2} = 78.54$, which is the area.

2. Find the area of a circle whose diameter is 7, and circumference 21.9912 yards.

Ans. 38.4846 *yds.*

3. How many square yards in a circle whose diameter is $3\frac{1}{2}$ feet, and circumference 10.9956?

Ans. 1.069016.

4. What is the area of a circle whose diameter is 100, and circumference 314.16?

Ans. 7854.

5. What is the area of a circle whose diameter is 1, and circumference 3.1416?

Ans. 0.7854.

6. What is the area of a circle whose diameter is 40, and circumference 131.9472?

Ans. 1319.472.

59. *How do you find the area of a circle when the diameter only is known?*

Square the diameter, and then multiply by the decimal .7854.

EXAMPLES.

1. What is the area of a circle whose diameter is 5?

We square the diameter, which gives us 25, and we then multiply this number and the decimal .7854 together.

Operation.

$$\begin{array}{rr} & .7854 \\ \overline{5}^2 = & 25 \\ \hline & 39270 \\ & 15708 \\ \hline \text{area} = & 19.6350 \\ \hline \end{array}$$

2. What is the area of a circle whose diameter is 7?

Ans. 38.4846.

3. What is the area of a circle whose diameter is 4.5?

Ans. 15.90435.

4. What is the number of square yards in a circle whose diameter is $1\frac{1}{6}$ yards?

Ans. 1.069016.

5. What is the area of a circle whose diameter is 8.75 feet?

Ans. 60.1322 *sq. ft.*

60. *How do you find the area of a circle when the circumference only is known?*

Multiply the square of the circumference by the decimal .07958, and the product will be the area very nearly.

EXAMPLES.

1. What is the area of a circle whose circumference is 3.1416?

We first square the circumference, and then multiply by the decimal .07958.

Operation.

$$\begin{array}{rr} \overline{3.1416}^2 = & 9.86965056 \\ & .07958 \\ \hline \text{area} = & .7854+ \\ \hline \end{array}$$

2. What is the area of a circle whose circumference is 91?

Ans. 659.00198.

3. Suppose a wheel turns twice in tracking $16\frac{1}{2}$ feet, and that it turns just 200 times in going round a circular bowling-green: what is the area in acres, roods, and perches?

Ans. 4 *A.* 3 *R.* 35.8 *P.*

4. How many square feet are there in a circle whose circumference is 10.9956 yards?

Ans. 86.5933.

5. How many perches are there in a circle whose circumference is 7 miles?

Ans. 399300.608.

61. *Having given a circle, how do you find a square which shall have an equal area?*

1st. The diameter $\times$ .8862 $=$ side of an equivalent square.

2d. The circumference $\times$ .2821 $=$ side of an equivalent square.

EXAMPLES.

1. The diameter of a circle is 100: what is the side of a square of an equal area?

Ans. 88.62.

2. The diameter of a circular fish-pond is 20 feet: what would be the side of a square fish-pond of an equal area?

Ans. 17.724 *ft.*

3. A man has a circular meadow, of which the diameter is 875 yards, and wishes to exchange it for a square one of equal size: what must be the side of the square?

Ans. 775.425.

4. The circumference of a circle is 200: what is the side of a square of an equal area?

Ans. 56.42.

5. The circumference of a round fish-pond is 400 yards: what is the side of a square fish-pond of equal area?

Ans. 112.84.

6. The circumference of a circular bowling-green is 412 yards: what is the side of a square one of equal area?

Ans. 116.2252 *yds.*

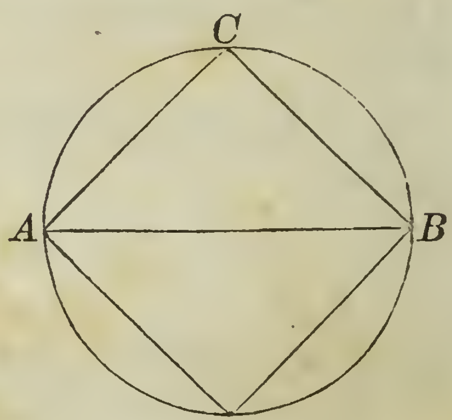
62. *Having given the diameter or circumference of a circle, how do you find the side of the inscribed square?*

1st. The diameter $\times$ .7071 $=$ side of the inscribed square.

2d. The circumference $\times$ .2251 $=$ side of the inscribed square.

EXAMPLES.

1. The diameter AB of a circle is 400: what is the value of AC, the side of the inscribed square?



Here

$.7071 \times 400 = 282.8400 = AC.$

2. The diameter of a circle is 412 feet: what is the side of the inscribed square?

Ans. 291.3252 *ft.*

3. If the diameter of a circle be 600, what is the side of the inscribed square?

Ans. 424.26.

4. The circumference of a circle is 312 feet: what is the side of the inscribed square?

Ans. 70.2312 *ft.*

5. The circumference of a circle is 819 yards: what is the side of the inscribed square?

Ans. 184.3569 *yds.*

6. The circumference of a circle is 715: what is the side of the inscribed square?

Ans. 160.9465.

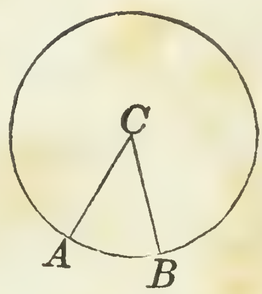
63. *How do you find the area of a circular sector?*

1st. Find the length of the arc by Art. **56**.

2d. Multiply the arc by one-half the radius, and the product will be the area.

EXAMPLES.

1. What is the area of the circular sector ACB, the arc AB containing 18°, and the radius CA being equal to 3 feet?



First, $.01745 \times 18 \times 3 = .94230 =$ length AB.

Then, $.94230 \times 1\frac{1}{2} = 1.41345 =$ area.

2. What is the area of a sector of a circle, in which the radius is 20 and the arc one of 22 degrees?

Ans. 76.7800.

3. Required the area of a sector whose radius is 25 and the arc one of 147° 29′.

Ans. 804.2448.

4. Required the area of a semicircle in which the radius is 13.

Ans. 265.4143.

5. What is the area of a circular sector when the length of the arc is 650 feet and the radius 325?

Ans. 105625 *sq. ft.*

64. *How do you find the area of a segment of a circle?*

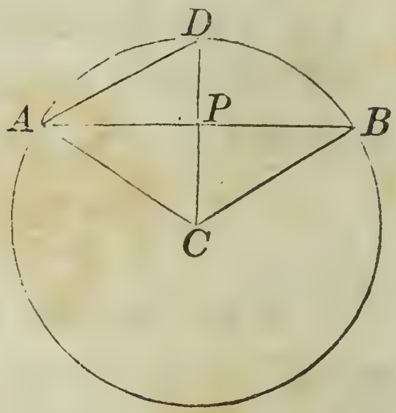
1st. Find the area of the sector having the same arc with the segment, by the last problem.

2d. Find the area of the triangle formed by the chord of the segment and the two radii through its extremities.

3d. If the segment is greater than the semicircle, add the two areas together; but if it is less, subtract them, and the result in either case will be the area required.

EXAMPLES.

1. What is the area of the segment ADB, the chord $AB = 24$ feet, and $CA = 20$ feet?



First, $CP = \sqrt{\overline{CA}^2 - \overline{AP}^2}$

$= \sqrt{400 - 144} = 16$;

then,

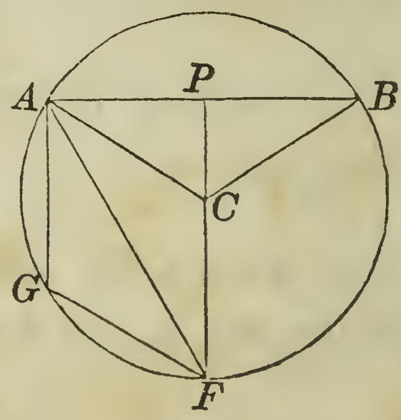
$PD = CD - CP = 20 - 16 = 4$,

and, $AD = \sqrt{\overline{AP}^2 + \overline{PD}^2} = \sqrt{144 + 16} = 12.64911$:

then, $\text{arc } ADB = \dfrac{12.64911 \times 8 - 24}{3} = 25.7309$.

arc	$ADB = 25.7309$	$AP =$	12
half radius	$= \quad 10$	$CP =$	16
area sector	$ADBC = 257.309$	area $CAB =$	192
area	$CAB = 192$		
	65.309 = area of segment ADB.		

2. Find the area of the segment AFB, knowing the following lines, viz.: $AB = 20.5$; $FP = 17.17$; $AF = 20$; $FG = 11.5$, and $CA = 11.64$.



Arc $AGF = \dfrac{FG \times 8 - AF}{3} = \dfrac{11.5 \times 8 - 20}{3} = 24$;

sector $AGFBC = 24 \times 11.64 = 279.36$:

but $CP = FP - AC = 17.17 - 11.64 = 5.53$:

Then, area $ACB = \dfrac{AB \times CP}{2} = \dfrac{20.5 \times 5.53}{2} = 56.6825$.

Then, area of sector $AFBC = 279.36$
do. of triangle $ABC = \underline{56.6825}$
gives area of segment $AFB = \underline{336.0425}$

3. What is the area of a segment, the radius of the circle being 10, and the chord of the arc 12 yards?

Ans. 16.324 *sq. yds.*

4. Required the area of the segment of a circle whose chord is 16, and the diameter of the circle 20.

Ans. 44.5903.

5. What is the area of a segment whose arc is a quadrant—the diameter of the circle being 18?

Ans. 63.6174.

6. The diameter of a circle is 100, and the chord of the segment 60: what is the area of the segment?

Ans. 408, nearly.

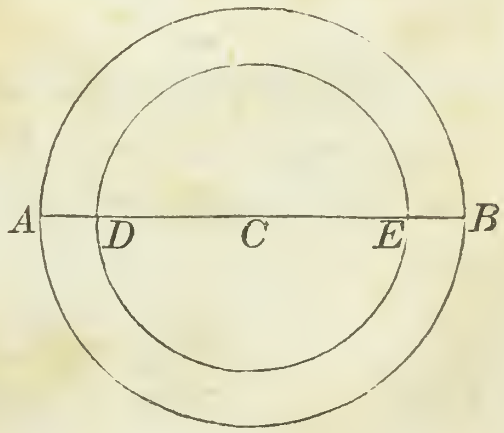
65. *How do you find the area of a circular ring; that is, the area included between the circumferences of two circles, having a common centre?*

1st. Square the diameter of each ring, and subtract the square of the less from that of the greater.

2d. Multiply the difference of the squares by the decimal .7854, and the product will be the area.

EXAMPLES.

1. In the concentric circles having the common centre C, we have $AB = 10$ yards, and $DE = 6$ yards: what is the area of the space included between them?



$$\overline{AB}^2 = \overline{10}^2 = 100$$
$$\overline{DE}^2 = 6^2 = 36$$
$$\text{Difference} = 64$$

Then, $64 \times .7854 = 50.2656 =$ area.

2. What is the area of the ring when the diameters of the circles are 20 and 10?

Ans. 235.62.

3. If the diameters are 20 and 15, what will be the area included between the circumferences?

Ans. 137.445.

4. If the diameters are 16 and 10, what will be the area included between the circumferences?

Ans. 122.5224.

5. Two diameters are 21.75 and 9.5; required the area of the circular ring.

Ans. 300.6609.

6. If the two diameters are 4 and 6, what is the area of the ring?

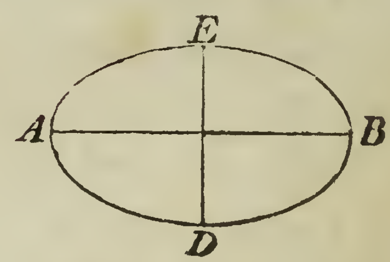
Ans. 15.708.

66. *How do you find the area of an ellipse?*

Multiply the two axes together, and their product by the decimal .7854, and the result will be the required area.

EXAMPLES.

1. Required the area of an ellipse, whose transverse axis $AB = 70$ feet, and the conjugate axis $DE = 50$ feet.



$$AB \times DE = 70 \times 50 = 3500:$$

Then, $.7854 \times 3500 = 2748.9 =$ area.

2. Required the area of an ellipse whose axes are 24 and 18.

Ans. 339.2928.

3. What is the area of an ellipse whose axes are 35 and 25?

Ans. 687.225.

4. What is the area of an ellipse whose axes are 80 and 60?

Ans. 3769.92.

5. What is the area of an ellipse whose axes are 50 and 45?

Ans. 1767.15.

SECTION II.

MENSURATION OF SOLIDS.

1. *What is a solid or body?*

A solid or body is that which has length, breadth, and thickness.

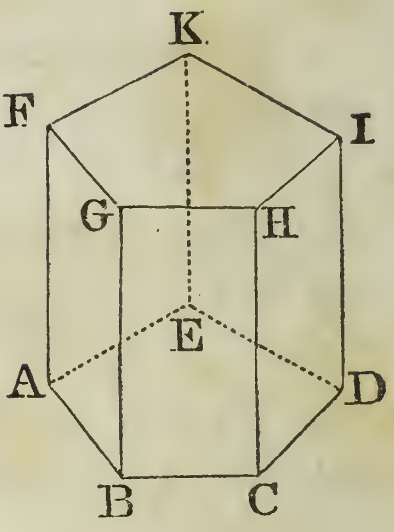
2. *What is a body bounded by planes called?*

Every solid bounded by planes is called a *polyedron*.

3. *What are the bounding planes, the straight lines, and the angular points called?*

The planes which bound a polyedron are called *faces*. The straight lines in which the faces intersect each other, are called the *edges* of the polyedron; and the points at which the edges intersect, are called the *vertices* of the angles, or vertices of the polyedron.

4. *What is a prism? What are its bases? what its convex surface?*



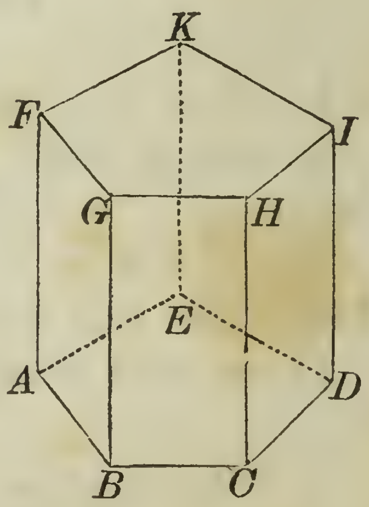
A prism is a solid, whose ends are equal polygons, and whose side faces are parallelograms.

Thus, the prism whose lower base is the pentagon $ABCDE$, terminates in an equal and parallel pentagon $FGHIK$, which is called the *upper base.* The side faces of the prism are the parallelograms DH, DK, EF, AG, BH. These are called the *convex* or *lateral* surface of the prism.

5. *What is the altitude of a prism?*

The altitude of a prism is the distance between its upper and lower bases; that is, it is a line drawn from a point of the upper base, perpendicular to the lower base.

6. *What is a right prism?*



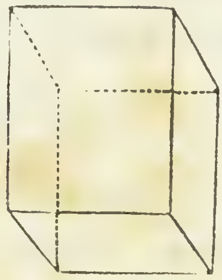
A right prism is one in which the edges AF, BG, EK, HC, and DI are perpendicular to the bases. In the right prism, either of the perpendicular edges is equal to the altitude. In the oblique prism the altitude is less than the edge.

7. *How are prisms distinguished from each other?*

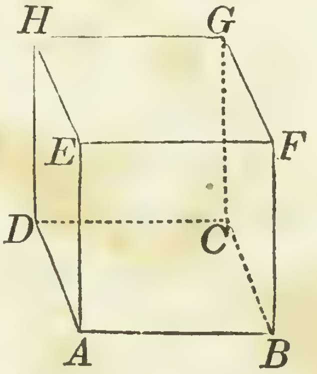
A prism whose base is a triangle, is called a *triangular* prism: if the base is a quadrangle, it is called a quadrangular prism: if a pentagon, a pentagonal prism: if a hexagon, it is called a hexagonal prism: &c.

8. *What is a parallelopipedon? what a cube?*

A prism whose base is a parallelogram, and all of whose faces are also parallelograms, is called a parallelopipedon. If all the faces are rectangles, it is called a rectangular parallelopipedon. If all the faces are squares, it is called a *cube*. The cube is bounded by six equal faces at right angles to each other.



9. *How do the opposite faces of a parallelopipedon compare with each other?*

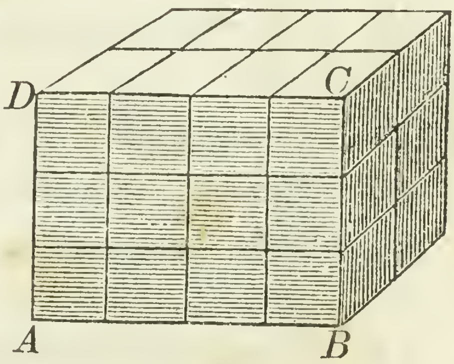


The opposite faces of a parallelopipedon are equal to each other. Thus, the parallelogram BD is equal to the opposite parallelogram FH, the parallelogram BE to CH, and BG to AH.

10. *What is the content of a solid?*

The *content* of a solid is the number of cubes which it contains.

In order to find the content of a solid, suppose $ABCD$ to be the base of a parallelopipedon.



Let us suppose $AB = 4$ feet, and $BC = 3$ feet. Then the number of square feet in the base will be equal to $3 \times 4 = 12$ square feet. Therefore, 12 equal cubes of one foot each, may be placed by the side of each other on the base. If the parallelopipedon be 1 foot in height, it will contain 12 such cubes, or 12 cubic feet: were it 2 feet in height, it would contain two tiers of cubes, or 24 cubic feet:

were it 3 feet in height, it would contain three tiers of cubes, or 36 cubic feet. Therefore, *the solid content of a parallelopipedon is equal to the product of its length, breadth, and height.*

11. *How many kinds of quantity are there in geometry?*

There are three kinds of quantity in geometry, viz.: Lines, Surfaces, and Solids; and each of these has its own unit.

12. *What are the units of these kinds of quantity?*

The unit of a line, which we have called the linear unit, is a line of a known length, as a foot, a yard, a rod, &c.

The unit of surface is a square, whose sides are the unit of length.

The unit of solidity is a cube, whose edges are the unit of length.

For example, if the bounding lines of a surface be estimated in yards, the content will be square yards; and if the bounding lines of a solid be yards, its surface will be estimated in square yards, and its solid content in cubic yards.

13. *Into how many parts is the mensuration of solids divided?*

The mensuration of solids is divided into two parts:—

1st. The mensuration of the surfaces of solids: and

2dly. The mensuration of their solidities.

14. *How is a curved line to be treated?*

A curve line which is expressed by numbers is also referred to a unit of length, and its numerical value is the number of times which the line contains the unit.

If, then, we suppose the linear unit to be reduced to a

straight line, and a square constructed on this line, this square will be the unit of measure for curved surfaces.

15. *Repeat the table of solid measures.*

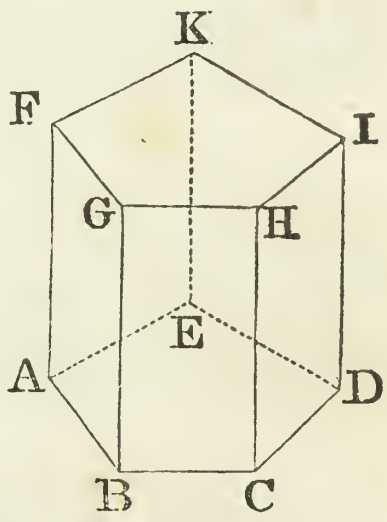
1 cubic foot	= 1728 cubic inches.
1 cubic yard	= 27 cubic feet.
1 cubic rod	= $4492\frac{1}{8}$ cubic feet.
1 ale gallon	= 282 cubic inches.
1 wine gallon	= 231 cubic inches.
1 bushel	= 2150.42 cubic inches.

OF THE PRISM.

16. *How do you find the surface of a right prism?*

Multiply the perimeter of the base by the altitude, and the product will be the convex surface: and to this add the area of the bases when the entire surface is required.

EXAMPLES.



1. Find the entire surface of the regular prism, whose base is the regular polygon $ABCDE$, and altitude AF, when each side of the base is 20 feet, and the altitude AF 50 feet.

$$AB + BC + CD + DE + EA = 100; \text{ and } AF = 50:$$

then $(AB + BC + CD + DE + EA) \times AF =$ convex surface becomes $100 \times 50 = 5000$ square feet, which is the convex surface. For the area of the end, we have $\overline{AB}^2 \times$ tabular number $=$ area $ABCDE$, (see page 131;) that is, $\overline{20}^2 \times$ tabular number, or $400 \times 1.720477 = 688.1908 =$ the area $ABCDE$.

Then, convex surface =	5000	square feet.
lower base	688.1908	do.
upper base	688.1908	do.
entire surface	6376.3816	do.

2. What is the surface of a cube, the length of each side being 20 feet?

Ans. 2400 *sq. ft.*

3. Find the entire surface of a triangular prism, whose base is an equilateral triangle, having each of its sides equal to 18 inches, and altitude 20 feet.

Ans. 91.949 *sq. ft.*

4. What is the convex surface of a regular octagonal prism, the side of whose base is 15 and altitude 12 feet?

Ans. 1440 *sq. ft.*

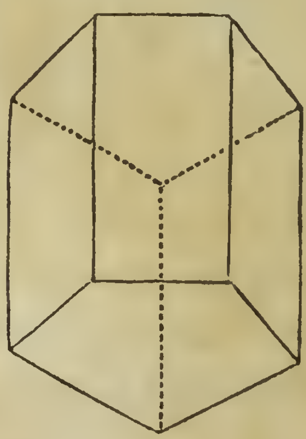
5. What must be paid for lining a rectangular cistern with lead at 2*d.* a pound, the thickness of the lead being such as to require 7*lb.* for each square foot of surface: the inner dimensions of the cistern being as follows: viz., the length 3 feet 2 inches, the breadth 2 feet 8 inches, and the depth 2 feet 6 inches?

Ans. £2 3*s.* $10\frac{5}{9}$.

17. *How do you find the solidity of a prism, parallelopipedon, or cube?*

Multiply the area of the base by the perpendicular height, and the product will be the solidity.

EXAMPLES.



1. What is the solidity of a regular pentagonal prism, whose altitude is 20, and each side of the base 15 feet?

To find the area of the base we have (page 131)

$\overline{15}^2 = 225$: and $225 \times 1.7204774 = 387.107415 =$ the area of the base: hence,

$$387.107415 \times 20 = 7742.1483 = \text{solidity.}$$

2. What is the solid content of a cube whose side is 24 inches?

Ans. 13824 *solid in.*

3. How many cubic feet in a block of marble, of which the length is 3 feet 2 inches, breadth 2 feet 8 inches, and height or thickness 2 feet 6 inches?

Ans. $21\frac{1}{9}$ *solid ft.*

4. How many gallons of water, ale measure, will a cistern contain whose dimensions are the same as in the last example?

Ans. $129\frac{17}{47}$.

5. Required the solidity of a triangular prism, whose altitude is 10 feet, and the three sides of its triangular base 3, 4, and 5 feet.

Ans. 60 *solid ft.*

6. What is the solidity of a square prism, whose height is $5\frac{1}{2}$ feet, and each side of the base $1\frac{1}{3}$ feet?

Ans. $9\frac{7}{9}$ *solid ft.*

7. What is the solidity of a prism, whose base is an equilateral triangle, each side of which is 4 feet, the height of the prism being 10 feet?

Ans. 69.282 *solid ft.*

8. What is the number of cubic or solid feet in a regular pentagonal prism, of which the altitude is 15 feet and each side of the base 3.75 feet?

Ans. 362.913.

9. What is the solidity of a prism, whose base is an equilateral triangle, each side of which is 1.5 feet, and the altitude 18 feet?

Ans. 17.53701435 *cubic ft.*

10. What is the solidity of a cube, whose side is 15 inches?

Ans. 1.953125 *cubic ft.*

11. What is the solidity of a cube, whose side is 17.5 inches?

Ans. 3.1015 *cubic ft.*

12. What is the solidity of a prism, whose base is a hexagon, each side of which is 1 foot 4 inches, and the length of the prism 15 feet?

Ans. 69.2820285 *solid ft.*

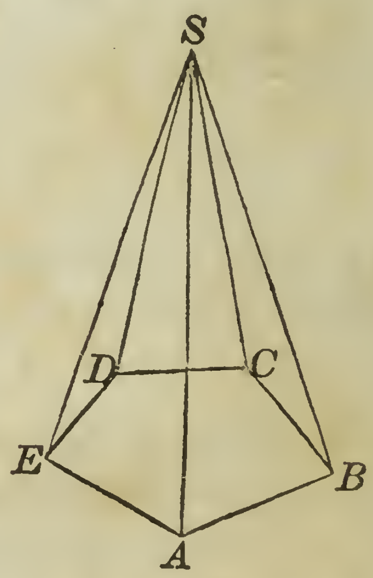
13. What is the solidity of a prism, whose altitude is 30 feet, and whose base is a heptagon, each side of which is 13 feet 3 inches?

Ans. 1913.936237175 *solid ft.*

OF THE PYRAMID.

18. *What is a pyramid, and what are its parts?*

A pyramid is a solid, formed by several triangles united at the same point *S*, and terminating in the different sides of a polygon *ABCDE*.



The polygon *ABCDE*, is called the base of the pyramid; the point *S* is called the *vertex*, and the triangles *ASB*, *BSC*, *CSD*, *DSE*, and *ESA*, form its *lateral*, or *convex* surface.

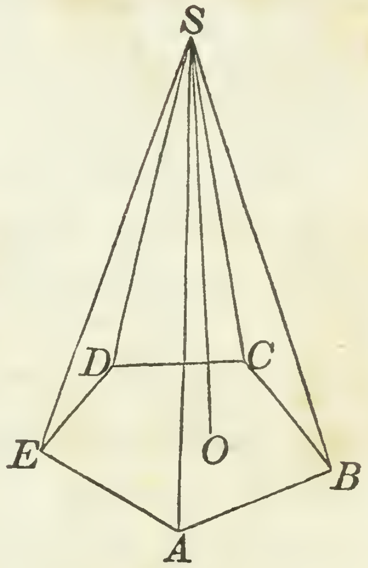
19. *What is a solid angle, and what is the least number of planes that can form one?*

A *solid angle* is the angular spaces included between several planes which meet at a point. Thus, the solid

angle S is formed by the meeting of the five planes ESD, DSC, CSB, BSA, and ASE. The point S is called the vertex of the solid angle. Three planes, at least, are required to form a solid angle.

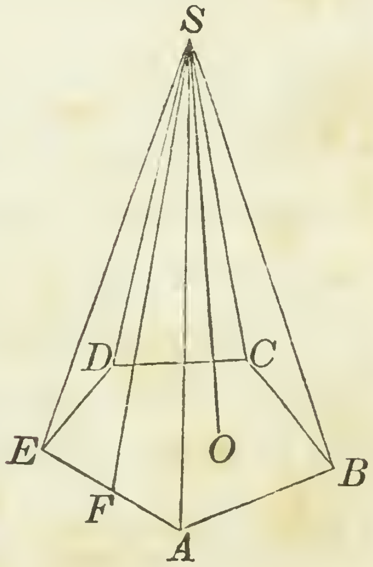
20. *What is the altitude of a pyramid?*

The *altitude* of a pyramid, is the perpendicular let fall from the vertex upon the plane of the base. Thus, SO is the altitude of the pyramid $S—ABCDE$.



21. *What is the slant height of a pyramid?*

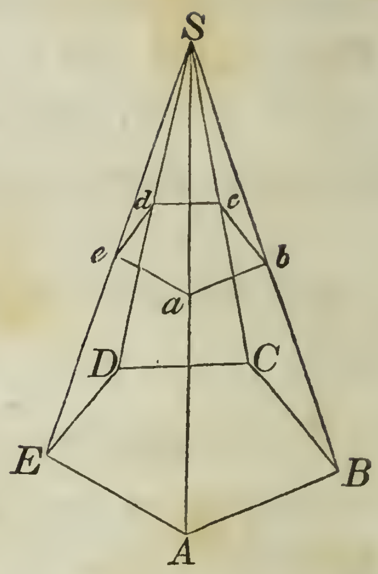
The *slant height* of a regular pyramid, is a line drawn from the vertex, perpendicular to one of the sides of the polygon which forms its base. Thus, SF is the slant height of the pyramid $S—ABCDE$.



22. *What is the axis of a pyramid?*

When the base of the pyramid is a regular polygon, and the perpendicular SO passes through the middle point of the base, the pyramid is called a regular pyramid, and the line SO is called the *axis*.

23. *What is the frustum of a pyramid? what the altitude of the frustum?*



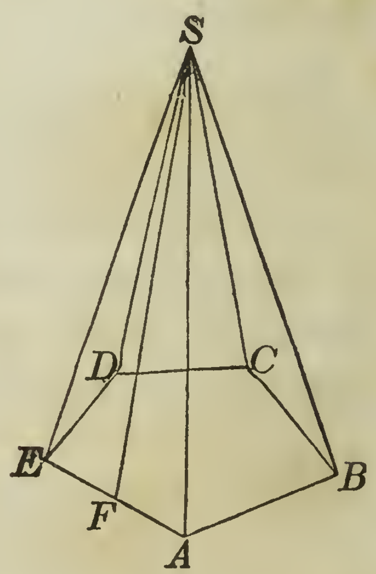
If from the pyramid *S—ABCDE* the pyramid *S—abcde* be cut off by a plane parallel to the base, the remaining solid, below the plane, is called the *frustum* of a pyramid.

The altitude of a frustum is the perpendicular distance between the upper and lower planes.

24. *How are pyramids distinguished?*

A pyramid whose base is a triangle, is called a triangular pyramid; if the base is a quadrangle, it is called a quadrangular pyramid; if a pentagon, it is called a pentagonal pyramid; if the base is a hexagon, it is called a hexagonal pyramid, &c.

25. *What is the convex surface of a regular pyramid equal to?*

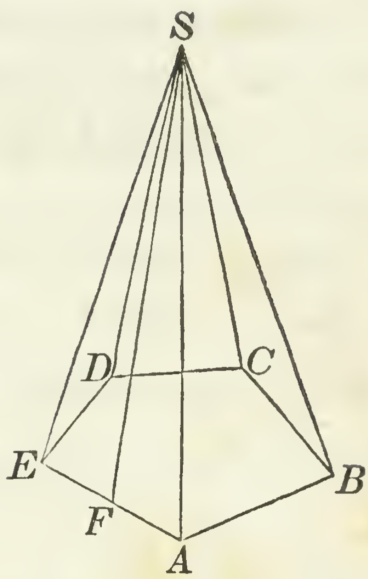


The convex surface of a regular pyramid, is equal to the perimeter of the base, multiplied by half the slant height. Thus, the convex surface of he pyramid *S—ABCDE* is equal to $\frac{1}{2}SF(AB+BC+CD+DE+EA.)$

26. *How do you find the surface of a regular pyramid?*

Multiply the perimeter of the base by half the slant height, and the product will be the convex surface: to this add the area of the base, if the entire surface is required.

EXAMPLES.



1. In the regular pentagonal pyramid $S—ABCDE$, the slant height SF is equal to 45, and each side of the base is 15 feet: required the convex surface, and also the entire surface.

$15 \times 5 = 75 =$ perimeter of the base,

$75 \times 22\frac{1}{2} = 1687.5$ square feet = area of convex surface.

And $\overline{15}^2 = 225$,

then $225 \times 1.7204774 = 387.107415 =$ the area of the base.

Hence, convex surface $= 1687.5$
area of the base $= 387.107415$
entire surface $= 2074.607415$ square feet.

2. What is the convex surface of a regular triangular pyramid, the slant height being 20 feet, and each side of the base 3 feet?

Ans. 90 *sq. ft.*

3. What is the entire surface of a regular pyramid, whose slant height is 15 feet, and the base a regular pentagon, of which each side is 25 feet?

Ans. 2012.798 *sq. ft.*

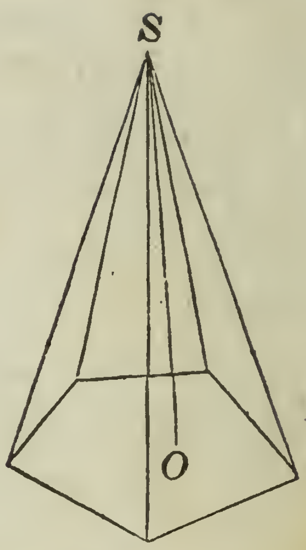
4. What is the entire surface of a regular octagonal pyramid, of which each side of the base is 9.941 yards, and the slant height 15?

Ans. 1073.628 *sq. yds.*

27. *How do you find the solidity of a pyramid?*

Multiply the area of the base by the altitude, and divide the product by three: the quotient will be the solidity.

EXAMPLES.



1. What is the solidity of a pyramid, the area of whose base is 215 square feet, and the altitude $SO = 45$ feet?

First, $215 \times 45 = 9675$:
then $9675 \div 3 = 3225$

which is the solidity expressed in solid feet.

2. Required the solidity of a square pyramid, each side of its base being 30, and its altitude 25.

Ans. 7500 *solid ft.*

3. How many solid yards are there in a triangular pyramid, whose altitude is 90 feet, and each side of its base 3 yards?

Ans. 38.97114.

4. How many solid feet in a triangular pyramid, the altitude of which is 14 feet 6 inches, and the three sides of its base 5, 6, and 7 feet?

Ans. 71.0352.

5. What is the solidity of a regular pentagonal pyramid, its altitude being 12 feet, and each side of its base 2 feet?

Ans. 27.5276 *solid ft.*

6. How many solid feet in a regular hexagonal pyramid, whose altitude is 6.4 feet, and each side of the base 6 inches?

Ans. 1.38564.

7. How many solid feet are contained in a hexagonal pyramid, the height of which is 45 feet, and each side of the base 10 feet?

Ans. 3897.1143.

8. The spire of a church is an octagonal pyramid, each side of the base being 5 feet 10 inches, and its perpendicular height 45 feet. Within is a cavity, or hollow part, each side of the base of which is 4 feet 11 inches, and its perpendicular height 41 feet: how many yards of stone does the spire contain?

Ans. 32.197353.

OF THE FRUSTUM OF A PYRAMID.

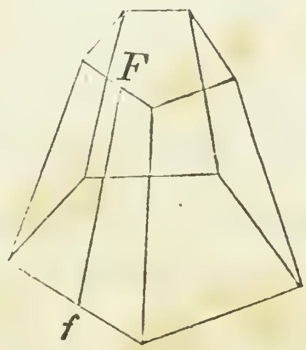
28. *How do you find the convex surface of the frustum of a regular pyramid?*

Multiply half the sum of the perimeters of the two bases by the slant height of the frustum, and the product will be the convex surface.

EXAMPLES.

1. In the frustum of the regular pentagonal pyramid each side of the lower base is 30 and each side of the upper base is 20 feet, and the slant height fF is equal to 15 feet. What is the convex surface of the frustum?

Ans. 1875 *sq. ft.*



2. How many square feet are there in the convex surface of the frustum of a square pyramid, whose slant height is 10 feet, each side of the lower base 3 feet 4 inches, and each side of the upper base 2 feet 2 inches?

Ans. 110.

3. What is the convex surface of the frustum of a heptagonal pyramid whose slant height is 55 feet, each side of the lower base 8 feet, and each side of the upper base 4 feet?

Ans. 2310 *sq. ft.*

29. *How do you find the entire surface of the frustum of a regular pyramid?*

To the convex surface, found as above, add the areas of the two ends, and the result will be the entire surface.

What is the entire surface of the frustum in each of the last three examples?

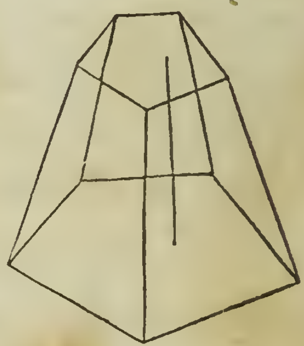
$$\textit{Ans.}\ \begin{cases} \text{1st. } 4111.62062\ \textit{sq. ft.} \\ \text{2d. } 125\tfrac{29}{36}\ \textit{sq. ft.} \\ \text{3d. } 2600.712992\ \textit{sq. ft.} \end{cases}$$

30. *How do you find the solidity of the frustum of a pyramid?*

Add together the areas of the two bases of the frustum and a geometrical mean proportional between them; and then multiply the sum by the altitude and take one-third of the product for the solidity.

EXAMPLES.

1. What is the solidity of the frustum of a pentagonal pyramid, the area of the lower base being 16 and of the upper base 9 square feet, the altitude being 7 feet?



First, $16 \times 9 = 144$: then $\sqrt{144} = 12$ the mean.

Then, area of lower base $= 16$

" upper base $= 9$

mean of bases $= 12$

$$\begin{array}{lr} & 37 \\ \text{height} & 7 \\ \hline & 3)\,259 \\ \hline \text{solidity} & = 86\tfrac{1}{3} \text{ solid feet.} \end{array}$$

2. What is the number of solid feet in a piece of timber whose bases are squares, each side of the lower base being 15 inches, and each side of the upper base being 6 inches—the length being 24 feet?

Ans. 19.5.

3. Required the solidity of a regular pentagonal frustum, whose altitude is 5 feet, each side of the lower base 18 inches, and each side of the upper base six inches.

Ans. 9.31925 *solid ft.*

4. What is the content of a regular hexagonal frustum, whose height is 6 feet, the side of the greater end 18 inches, and of the less end 12 inches?

Ans. 24.681724 *cubic ft.*

5. How many cubic feet in a square piece of timber, the areas of the two ends being 504 and 372 inches, and its length $31\frac{1}{2}$ feet?

Ans. 95.447.

6. What is the solidity of a square piece of timber, its length being 18 feet, each side of the greater base 18 inches, and each side of the smaller 12 inches?

Ans. 28.5 *cubic ft.*

7. What is the solidity of the frustum of a regular hexagonal pyramid, the side of the greater end being 3 feet, that of the less 2 feet, and the height 12 feet?

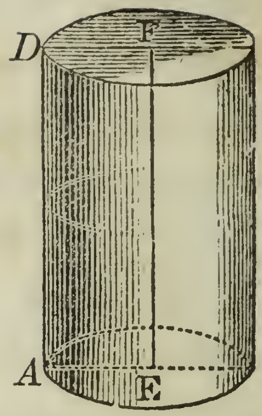
Ans. 197.453776 *solid ft.*

OF THE CYLINDER.

31. *What is a cylinder? what its upper, and what its lower base? What is the axis?*

A *Cylinder* is a solid, described by the revolution of a rectangle *AEFD*, about a fixed side *EF*.

As the rectangle $AEFD$ turns around the side EF, like a door upon its hinges, the lines AE and FD describe circles, and the line AD describes the *convex surface* of the cylinder.

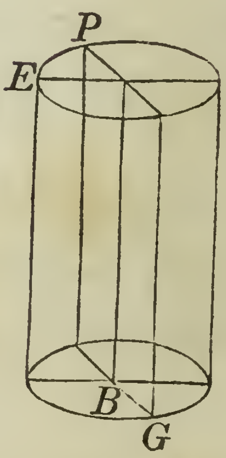


The circle described by the line AE is called the *lower base* of the cylinder, and the circle described by DF is called the *upper base.*

The immoveable line EF is called the *axis* of the cylinder.

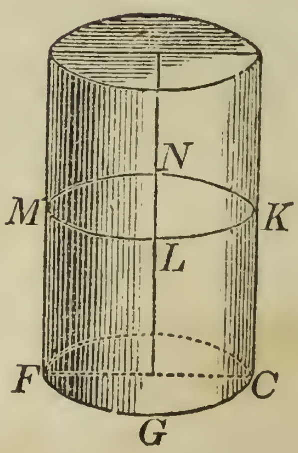
A cylinder, therefore, is a round body with circular ends.

32. *How will a plane, passed through the axis, cut the cylinder?*



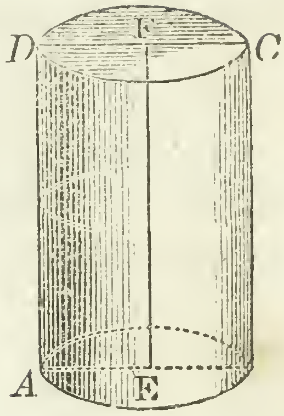
If a plane be passed through the axis of a cylinder, it will intersect it in a rectangle PG, which is double the revolving rectangle EB.

33. *If a cylinder be cut by a plane parallel to the base, how is the section?*



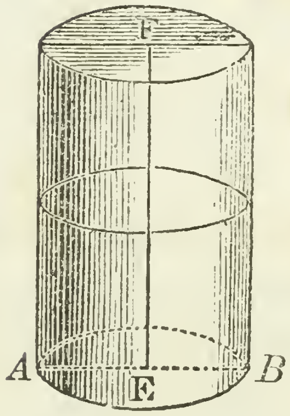
If a cylinder be cut by a plane parallel to the base, the section will be a circle equal to the base. Thus, $MLKN$ is a circle equal to the base FGC.

34. *How do you find the surface of a cylinder?*



The convex surface of a cylinder is equal to the circumference of the base, multiplied by the altitude. Thus, the convex surface of the cylinder AC, is equal to circumference of base $\times AD$: to this add the areas of the two ends, when the entire surface is required.

EXAMPLES.



1. What is the entire surface of the cylinder, in which AB, the diameter of the base, is 12 feet, and the altitude EF 30 feet?

First, to find the circumference of the base, (see page 133,) we have $3.1416 \times 12 = 37.6992 =$ circumference of the base.

Then, $37.6992 \times 30 = 1130.9760 =$ convex surface.

Also, $\overline{12}^2 = 144$: and $144 \times .7854 = 113.0976 =$ area of the base.

Then,	convex surface	$= 1130.9760$
	lower base	113.0976
	upper base	113.0976
	Entire area	$= 1357.1712$

2. What is the convex surface of a cylinder, the diameter of whose base is 20, and altitude 50 feet?

Ans. 3141.6 *sq. ft.*

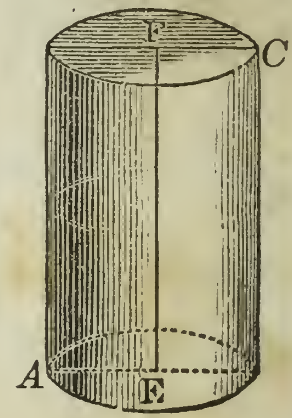
3. Required the entire surface of a cylinder, whose altitude is 20 feet, and the diameter of the base 2 feet.

Ans. 131.9472 *ft.*

4. What is the convex surface of a cylinder, the diameter of whose base is 30 inches, and altitude 5 feet?

Ans. 5654.88 *sq. inches.*

5. Required the convex surface of a cylinder, whose altitude is 14 feet, and the circumference of the base 8 feet 4 inches. *Ans.* 116.6666, &c., *sq. ft.*

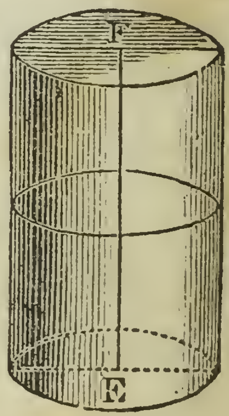


35. *How do you find the solidity of a cylinder?*

The solidity of a cylinder is equal to the area of the base, multiplied by the altitude. Thus, the solidity of the cylinder AC is equal to

area of base $\times$ FE.

EXAMPLES.



1. What is the solidity of a cylinder, the diameter of whose base is 40 feet, and altitude EF, 25 feet?

First, to find the area of the base, we have, (see page 137,)

$\overline{40}^2 = 1600$, then $1600 \times .7854 =$
$1256.64 =$ area of the base. Then,
$1256.64 \times 25 = 31416$ solid feet,
which is the solidity.

2. What is the solidity of a cylinder, the diameter of whose base is 30 feet, and altitude 50 feet?

Ans. 35343 *cubic ft.*

3. What is the solidity of a cylinder whose height is 5 feet, and the diameter of the end 2 feet?

Ans. 15.708 *solid ft.*

4. What is the solidity of a cylinder whose height is 20 feet, and the circumference of the base 20 feet?

Ans. 636.64 *cubic ft.*

5. The circumference of the base of a cylinder is 20 feet, and the altitude 19.318 feet: what is the solidity?

Ans. 614.93 *cubic ft.*

6. What is the solidity of a cylinder whose altitude is 12 feet, and the diameter of its base 15 feet?

Ans. 2120.58 *cubic ft.*

7. Required the solidity of a cylinder whose altitude is 20 feet, and the circumference of whose base is 5 feet 6 inches. *Ans.* 48.1459 *cubic ft.*

8. What is the solidity of a cylinder, the circumference of whose base is 38 feet, and altitude 25 feet?

Ans. 2872.838 *cubic ft.*

9. What is the solidity of a cylinder, the circumference of whose base is 40 feet, and altitude 30 feet?

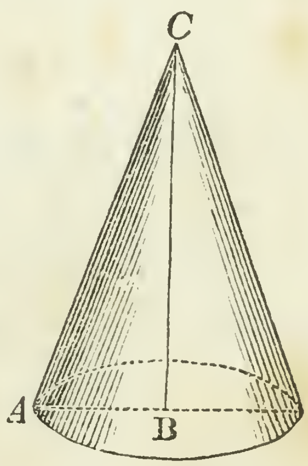
Ans. 3819.84 *solid ft.*

10. The diameter of the base of a cylinder is 84 yards, and the altitude 21 feet: how many solid or cubic yards does it contain? *Ans.* 38792.4768.

OF THE CONE.

36. *How is a cone described? What is its base? what its convex surface? what its altitude, and what its vertex?*

A cone is a solid, described by the revolution of a right-angled triangle ABC, about one of its sides CB.

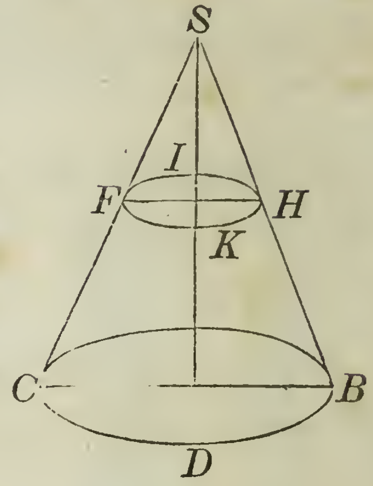


The circle described by the revolving side AB, is called the *base* of the cone.

The hypothenuse AC is called the *slant height of the cone*, and the surface described by it is called the *convex surface* of the cone.

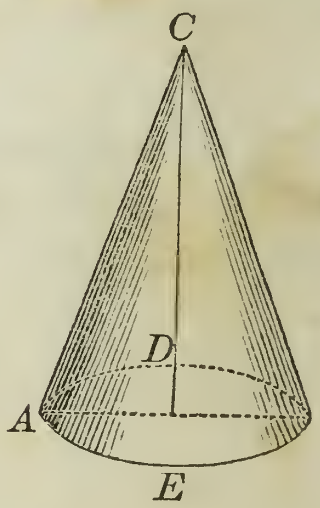
The side of the triangle CB, which remains fixed, is called the *axis* or *altitude* of the cone, and the point C the *vertex* of the cone.

37. *What is the frustum of a cone?*



If a cone be cut by a plane parallel to the base, the section will be a circle. Thus, the section $FKHI$ is a circle. If from the cone $S—CDB$, the cone $S—FKH$ be taken away, the remaining part is called the *frustum* of a cone.

38. *How do you find the surface of a cone?*

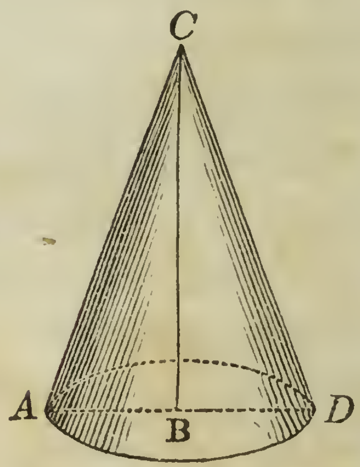


The convex surface of a cone is equal to the circumference of the base multiplied by half the slant height. Thus, the convex surface of the cone $C—AED$ is equal to

$$\text{circumference } AED \times \tfrac{1}{2} CA:$$

to this add the area of the base, when the entire surface is required.

EXAMPLES.



1. What is the convex surface of the cone whose vertex is C, the diameter AD of its base being $8\frac{1}{2}$ feet, and the side CA 50 feet?

First, $3.1416 \times 8\frac{1}{2} = 26.7036 =$ circum. of base.

Then, $\dfrac{26.7036 \times 50}{2} = 667.59 =$ convex surface.

2. Required the entire surface of a cone, whose side is 36, and the diameter of its base 18 feet.

Ans. 1272.348 *sq. ft.*

3. The diameter of the base is 3 feet, and the slant height 15 feet: what is the convex surface of the cone?

Ans. 70.686 *sq. ft.*

4. The diameter of the base of a cone is 4.5 feet, and the slant height 20 feet: what is the entire surface?

Ans. 157.27635 *sq. ft.*

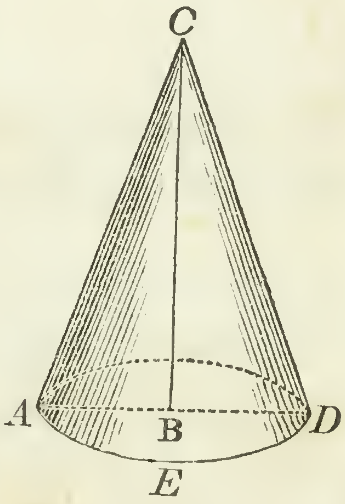
5. The circumference of the base of a cone is 10.75 and the slant height is 18.25: what is the entire surface?

Ans. 107.29021 *sq. ft.*

39. *How do you find the solidity of a cone?*

The solidity of a cone is equal to the area of the base multiplied by one-third of the altitude. Thus, the solidity of the cone C—AED is equal to

base $AED \times \frac{1}{3}CB$.

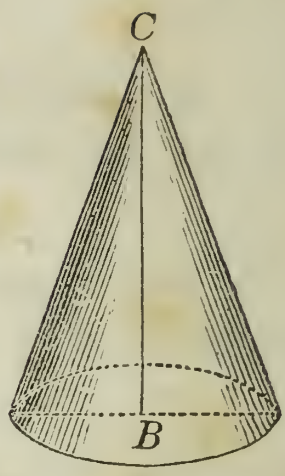


40. *How do a cone and cylinder, of the same base and altitude, compare with each other?*

Since the solidity of a cylinder is equal to the base multiplied by the altitude, and that of a cone to the base multiplied by one-third of the altitude, it follows that if a cylinder and cone have equal bases and altitudes, the *cone will be one-third of the cylinder.*

EXAMPLES.

1. What is the solidity of a cone, the area of whose base is 380 square feet, and altitude CB 48 feet?



We simply multiply the area of the base by the altitude, and then divide the product by 3.

Operation.

$$\begin{array}{r} 380 \\ 48 \\ \hline 3040 \\ 1520 \\ \hline 3)18240 \\ \hline \text{area} = 6080 \end{array}$$

2. Required the solidity of a cone whose altitude is 27 feet, and the diameter of the base 10 feet.

Ans. 706.86 *cubic ft.*

3. Required the solidity of a cone whose altitude is $10\frac{1}{2}$ feet, and the circumference of its base 9 feet.

Ans. 22.5609 *cubic ft.*

4. What is the solidity of a cone, the diameter of whose base is 18 inches, and altitude 15 feet?

Ans. 8.83575 *cubic ft.*

5. The circumference of the base of a cone is 40 feet, and the altitude 50 feet: what is the solidity?

Ans. 2122.1333 *solid ft.*

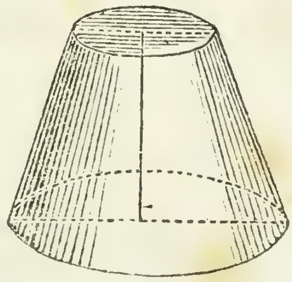
OF THE FRUSTUM OF A CONE.

41. *How do you find the surface of the frustum of a cone?*

Add together the circumferences of the two bases, and multiply the sum by half the slant height of the frustum; the product will be the convex surface, to which add the areas of the bases, when the entire surface is required.

EXAMPLES.

1. What is the convex surface of the frustum of a cone, of which the slant height is $12\frac{1}{2}$ feet, and the circumferences of the bases 8.4 and 6 feet?



We merely take the sum of the circumferences of the bases, and multiply by half the slant height.

Operation.

$$\begin{array}{rr} & 8.4 \\ & \underline{6} \\ & 14.4 \\ \text{half side} & \underline{6.25} \\ \text{area} = & \underline{90}\ \textit{sq. ft.} \end{array}$$

2. What is the entire surface of the frustum of a cone, the side being 16 feet, and the radii of the bases 2 and 3 feet?

Ans. 292.1688 *sq. ft.*

3. What is the convex surface of the frustum of a cone, the circumference of the greater base being 30 feet, and of the less 10 feet; the slant height being 20 feet?

Ans. 400 *sq. ft.*

4. Required the entire surface of the frustum of a cone whose slant height is 20 feet, and the diameters of the bases 8 and 4 feet.

Ans. 439.824 *sq. ft.*

5. A cone whose slant height is 30 feet, and the circum ference of its base 10 feet, is cut by a plane 6 feet from the vertex, measured on the slant height: what is the convex surface of the frustum?

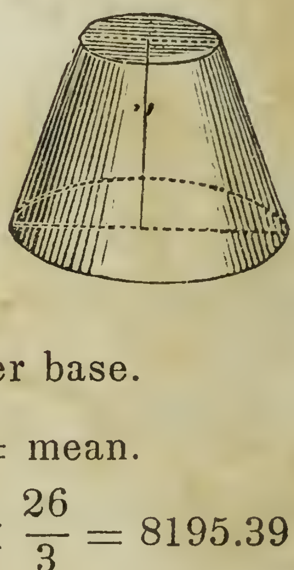
Ans. 144 *sq. ft.*

42. *How do you find the solidity of the frustum of a cone?*

1st. Add together the areas of the two ends and a geometrical mean between them.

2d. Multiply this sum by one-third of the altitude, and the product will be the solidity.

EXAMPLES.



1. How many cubic feet in the frustum of a cone, whose altitude is 26 feet, and the diameters of the bases 22 and 18 feet?

First, $\overline{22}^2 \times .7854 = 380.134 =$ area of lower base:

and $\overline{18}^2 \times .7854 = 254.47 =$ area of upper base.

Then, $\sqrt{380.134 \times 254.47} = 311.018 =$ mean.

Then, $(380.134 + 254.47 + 311.018) \times \frac{26}{3} = 8195.39$ which is the solidity.

2. How many cubic feet in a piece of round timber, the diameter of the greater end being 18 inches, and that of the less 9 inches, and the length 14.25 feet?

Ans. 14.68943.

3. What is the solidity of the frustum of a cone, the altitude being 18, the diameter of the lower base 8, and that of the upper base 4?

Ans. 527.7888.

4. What is the solidity of the frustum of a cone, the altitude being 25, the circumference of the lower base 20, and that of the upper base 10?

Ans. 464.216.

5. If a cask, which is composed of two equal conic frustums joined together at their larger bases, have its bung diameter 28 inches, the head diameter 20 inches, and the length 40 inches, how many gallons of wine will it contain, there being 231 cubic inches in a gallon?

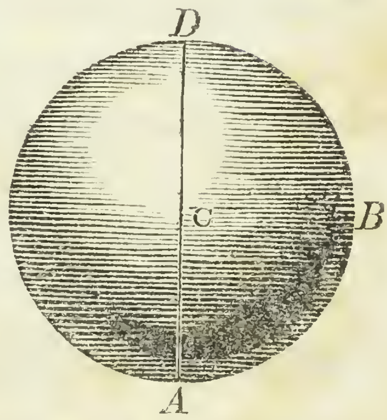
Ans. 79.0613.

OF THE SPHERE.

43. *What is a sphere?*

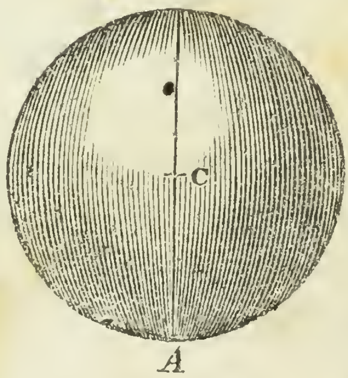
A *sphere* is a solid terminated by a curved surface, all the points of which are equally distant from a certain point within called the centre.

44. *How may a sphere be described?*



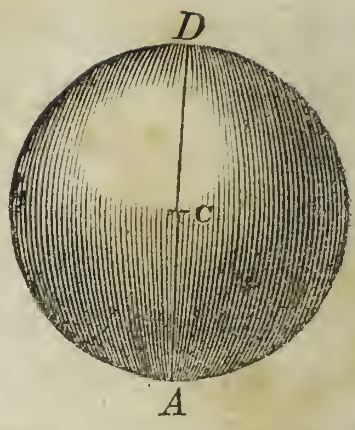
The sphere may be described by revolving a semicircle *ABD* about the diameter *AD*. The plane will describe the solid sphere, and the semi-circumference *ABD* will describe the surface.

45. *What is the radius of a sphere?*



The *radius* of a sphere is a line drawn from the centre to any point of the circumference. Thus, *CA* is a radius.

46. *What is the diameter of a sphere?*



The diameter of a sphere is a line passing through the centre, and terminated by the circumference. Thus, *AD* is a diameter.

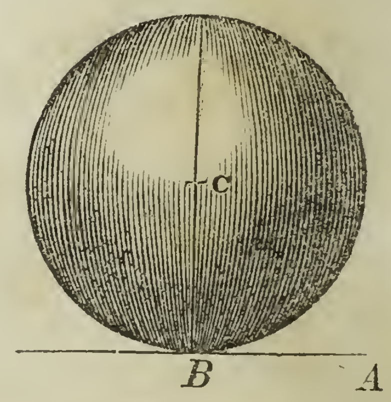
47. *How do the diameters of a sphere compare with each other?*

All diameters of a sphere are equal to each other; and each is double a radius.

48. *What is the axis of a sphere?*

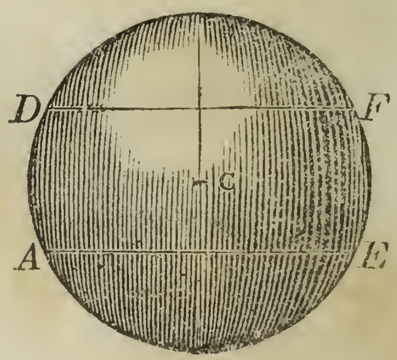
The axis of a sphere is any line about which it revolves; and the points at which the axis meets the surface, are called the *poles*.

49. *When is a plane said to be tangent to a sphere?*



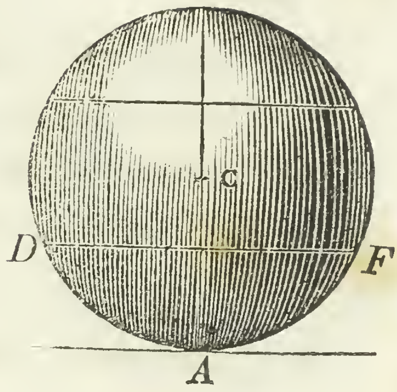
A *plane* is *tangent* to a sphere when it has but one point in common with it. Thus, *AB* is a tangent plane.

50. *What is a spherical zone? what are its bases?*



A *zone* is a portion of the surface of a sphere, included between two parallel planes which form its bases. Thus, the part of the surface included between the planes *AE* and *DF* is a zone. The bases of this zone are two circles whose diameters are *AE* and *DF*.

51. *When will a zone have but one base?*



One of the planes which bound a zone may become tangent to the sphere, in which case the zone will have but one base. Thus, if one plane be tangent to the sphere at A, and another plane cut it in the circle DF, the zone included between them will have but one base.

52. *What is a spherical segment?*

A *spherical segment* is a portion of the solid sphere included between two parallel planes. These parallel planes are its bases. If one of the planes is tangent to the sphere, the segment will have but one base.

53. *What is the altitude of a zone?*

The *altitude* of a zone or segment, is the distance between the parallel planes which form its bases.

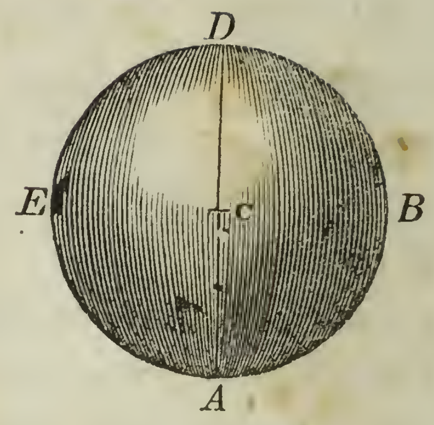
54. *How does a plane cut a sphere?*

Every plane passing through a sphere intersects the solid sphere in a circle, and the surface of the sphere in the circumference of a circle.

55. *When does a plane cut a sphere in a great circle? when in a small circle?*

If the intersecting plane passes through the centre of the sphere, the circle is called a *great circle.* If it does not pass through the centre, the circle of section is called a *small circle.*

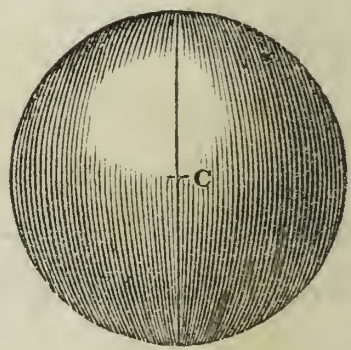
56. *How do you find the surface of a sphere?*



The surface of a sphere is equal to the product of its diameter by the circumference of a great circle. Thus, the surface of the sphere whose centre is C, is equal to

circumference $ABDE \times AD$.

EXAMPLES.



1. What is the surface of the sphere whose centre is C, the diameter being 7 feet?
Ans. 153.9384 *sq. ft.*

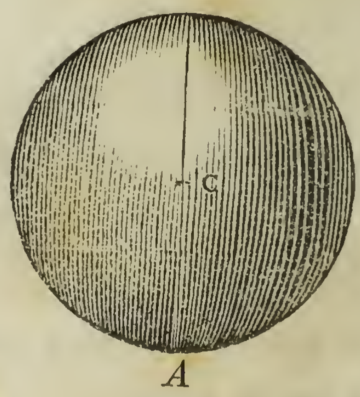
2. What is the surface of a sphere whose diameter is 24? *Ans.* 1809.5616.

3. Required the surface of a sphere whose diameter is 7921 miles. *Ans.* 197111024 *sq. miles.*

4. What is the surface of a sphere the circumference of whose great circle is 78.54? *Ans.* 1963.5.

5. What is the surface of a sphere whose diameter is $1\frac{1}{3}$ feet? *Ans.* 5.58506 *sq. ft.*

57. *How do you find the solidity of a sphere?*

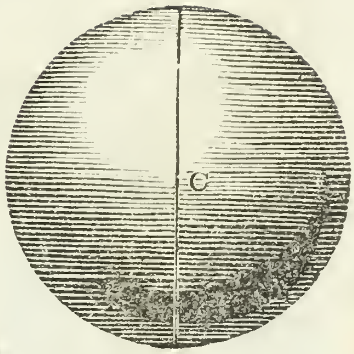


The solidity of a sphere is equal to its surface multiplied by one-third of the radius. Thus, the sphere whose centre is C, is equal to

surface $\times \frac{1}{3}\, CA$.

EXAMPLES.

1. What is the solidity of a sphere whose diameter is 12 feet?



First, $3.1416 \times 12 = 37.6992 =$ circumference of sphere,

$$\begin{array}{rcr} \text{diameter} & = & 12 \\ \hline \text{surface} & = & 452.3904 \\ \text{one-third radius} & = & 2 \\ \hline \text{solidity} & = & 904.7808 \text{ cubic feet.} \end{array}$$

2. The diameter of a sphere is 7957.8: what is its solidity?

Ans. 263863122758.4778.

3. The diameter of a sphere is 24 yards: what is its solid content?

Ans. 7238.2464 *cubic yds.*

4. The diameter of a sphere is 8: what is its solidity?

Ans. 268.0832.

58. *What is a second method of finding the solidity of a sphere?*

Cube the diameter and multiply the number thus found by the decimal .5236, and the product will be the solidity.

EXAMPLES.

1. What is the solidity of a sphere whose diameter is 20?

Ans. 4188.8.

2. What is the solidity of a sphere whose diameter is 6?

Ans. 113.0976.

3. What is the solidity of a sphere whose diameter is 10?

Ans. 523.6.

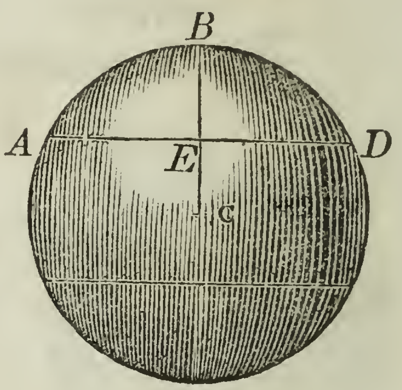
OF SPHERICAL ZONES.

59. *How do you find the convex surface of a spherical zone?*

Multiply the height of the zone by the circumference of a great circle of the sphere, and the product will be the convex surface.

EXAMPLES.

1. What is the convex surface of the zone ABD, the height BE being 9 inches, and the diameter of the sphere 42 inches?



First, $42 \times 3.1416 = 131.9472$ = circumference.
height $= 9$
surface $= 1187.5248$ square inches.

2. The diameter of a sphere is $12\frac{1}{2}$ feet: what will be the surface of a zone whose altitude is 2 feet?

Ans. 78.54 *sq. ft.*

3. The diameter of a sphere is 21 inches: what is the surface of a zone whose height is $4\frac{1}{2}$ inches?

Ans. 296.8812 *sq. in.*

4. The diameter of a sphere is 25 feet, and the height of the zone 4 feet: what is the surface of the zone?

Ans. 314.16 *sq. ft.*

OF SPHERICAL SEGMENTS.

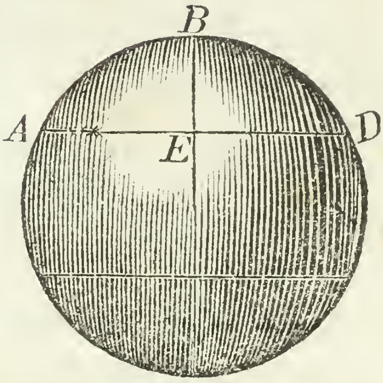
60. *How do you find the solidity of a spherical segment with one base?*

1st. To three times the square of the radius of the base add the square of the height.

2d. Multiply this sum by the height, and the product by the decimal .5236; the result will be the solidity of the segment.

EXAMPLES.

1. What is the solidity of the segment ABD, the height BE being 4 feet, and the diameter AD of the base being 14 feet?



First,

$(\overline{7}^2 \times 3 + \overline{4}^2) = 147 + 16 = 163.$

Then, $163 \times 4 \times .5236 = 341.3872$ solid feet, which is the solidity of the segment.

2. What is the solidity of the segment of a sphere, whose height is 4, and the radius of its base 8?

Ans. 435.6352.

3. What is the solidity of a spherical segment, the diameter of its base being 17.23368, and its height 4.5?

Ans. 572.5566.

4. What is the solidity of a spherical segment, the diameter of the sphere being 8, and the height of the segment 2 feet?

Ans. 41.888 *cubic ft.*

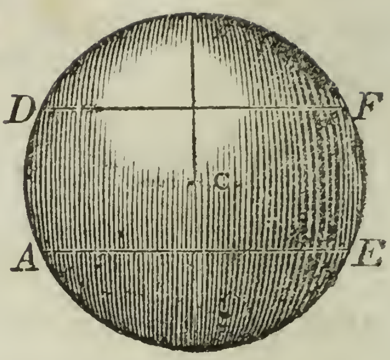
5. What is the solidity of a segment, when the diameter of the sphere is 20, and the altitude of the segment 9 feet?

Ans. 1781.2872 *cubic ft.*

61. *How do you find the solidity of a spherical segment having two bases?*

To the sum of the squares of the radii of the two bases add one-third of the square of the distance between them; then multiply this sum by the breadth, and the product by 1.5708, and the result will be the solidity.

EXAMPLES.



1. What is the solid content of the zone $ADFE$, the diameter of whose greater base DF is equal to 20 inches, and the less diameter AE 15 inches, and the distance between the two bases 10 inches?

Now, by the rule

$$[(10)^2 + (7.5)^2 + \frac{(10)^2}{3}] \times 10 \times 1.5708$$

$$= (100 + 56.25 + 33.33) \times 10 \times 1.5708$$

$$= 189.58 \times 10 \times 1.5708 = 2977.92264 \text{ solid inches.}$$

2. What is the solid content of a zone, the diameter of whose greater base is 24 inches, the less diameter 20 inches, and the distance between the bases 4 inches?

Ans. 1566.6112 *solid in.*

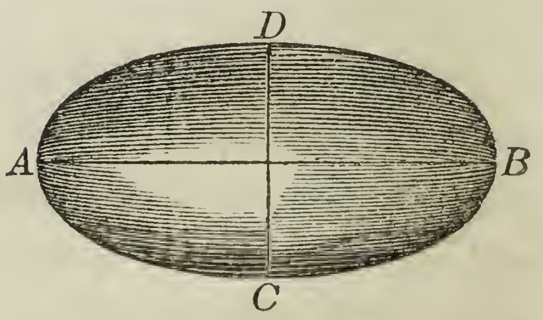
3. What is the solidity of the middle zone of a sphere, the diameter of whose bases are each 3 feet, and the distance between them 4 feet? *Ans.* 61.7848 *solid ft.*

OF THE SPHEROID.

62. *What is a spheroid?*

A spheroid is a solid, described by the revolution of an ellipse about either of its axes.

63. *What is the difference between a prolate and an oblate spheroid?*



If an ellipse $ACBD$ be revolved about the transverse or longer axis AB, the solid described is called a *prolate spheroid;* and if it be revolved about the shorter axis CD, the solid described is called an *oblate spheroid.*

64. *What is the form of the earth?*

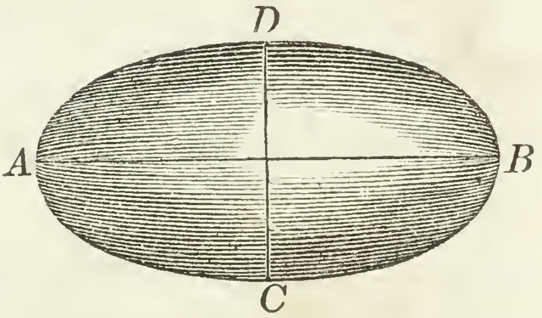
The earth is an oblate spheroid, the axis about which it revolves being about 34 miles shorter than the diameter perpendicular to it.

65. *How do you find the solidity of an ellipsoid?*

Multiply the fixed axis by the square of the revolving axis, and the product by the decimal .5236; the result will be the required solidity.

EXAMPLES.

1. In the prolate spheroid $ACBD$, the transverse axis $AB = 90$, and the revolving axis $CD = 70$ feet: what is the solidity?



Here, $AB = 90$ feet: $\overline{CD}^2 = \overline{70}^2 = 4900$: hence

$AB \times \overline{CD}^2 \times .5236 = 90 \times 4900 \times .5236 = 230907.6$ cubic feet, which is the solidity.

2. What is the solidity of a prolate spheroid, whose fixed axis is 100, and revolving axis 6 feet?

Ans. 1884.96.

3. What is the solidity of an oblate spheroid, whose fixed axis is 60, and revolving axis 100?

Ans. 314160.

4. What is the solidity of a prolate spheroid, whose axes are 40 and 50? *Ans.* 41888.

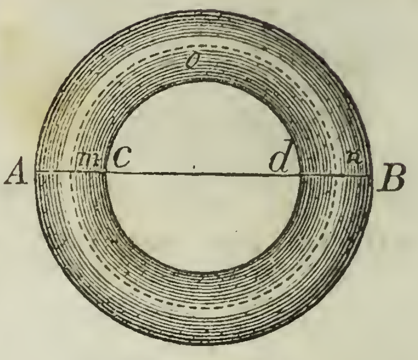
5. What is the solidity of an oblate spheroid, whose axes are 20 and 10? *Ans.* 2094.4.

6. What is the solidity of a prolate spheroid, whose axes are 55 and 33? *Ans.* 31361.022.

OF CYLINDRICAL RINGS.

66. *How is a cylindrical ring formed?*

A cylindrical ring is formed by bending a cylinder until the two ends meet each other. Thus, if a cylinder be bent round until the axis takes the position *mon*, a solid will be formed, which is called a cylindrical ring.



The line *AB* is called the outer, and *cd* the inner diameter.

67. *How do you find the convex surface of a cylindrical ring?*

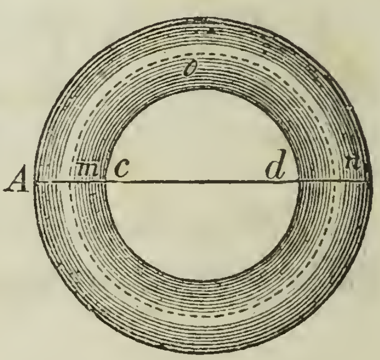
1st. To the thickness of the ring add the inner diameter.

2d. Multiply this sum by the thickness, and the product by 9.8696; the result will be the area.

EXAMPLES.

1. The thickness *Ac* of a cylindrical ring is 3 inches, and the inner diameter *cd* is 12 inches: what is the convex surface?

$Ac + cd = 3 + 12 = 15$: then $15 \times 3 \times 9.8696 = 444.132$ square inches $=$ the surface.



2. The thickness of a cylindrical ring is 4 inches, and the inner diameter 18 inches: what is the convex surface?

Ans. 868.52 *sq. in.*

3. The thickness of a cylindrical ring is 2 inches, and the inner diameter 18 inches: what is the convex surface?

Ans. 394.784 *sq. in.*

68. *How do you find the solidity of a cylindrical ring?*

1st. To the thickness of the ring add the inner diameter.

2d. Multiply this sum by the square of half the thickness, and the product by 9.8696; the result will be the required solidity.

EXAMPLES.

1. What is the solidity of an anchor-ring, whose inner diameter is 8 inches, and thickness in metal 3 inches? $8 + 3 = 11$: then, $11 \times (\frac{3}{2})^2 \times 9.8696 = 244.2726$, which expresses the solidity in cubic inches.

2. The inner diameter of a cylindrical ring is 18 inches, and the thickness 4 inches: what is the solidity of the ring?

Ans. 868.5248 *cubic in.*

3. Required the solidity of a cylindrical ring, whose thickness is 2 inches, and inner diameter 12 inches?

Ans. 138.1744 *cubic in.*

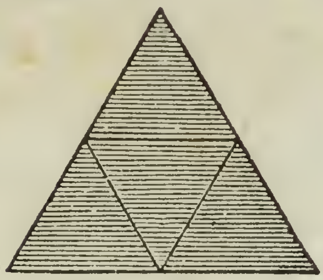
4. What is the solidity of a cylindrical ring, whose thickness is 4 inches, and inner diameter 16 inches?

Ans. 789.568 *cubic in.*

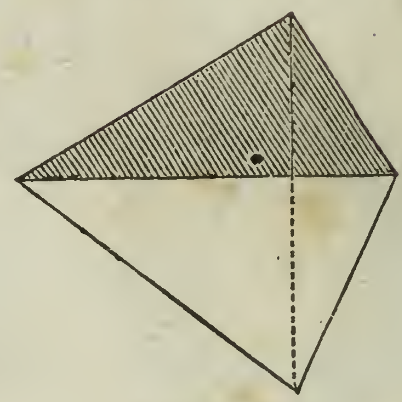
OF THE FIVE REGULAR SOLIDS.

69. A *regular solid* is one whose faces are all equal polygons, and whose solid angles are equal. There are five such solids.

1. The *tetraedron*, or equilateral pyramid, is a solid bounded by four equal triangles.

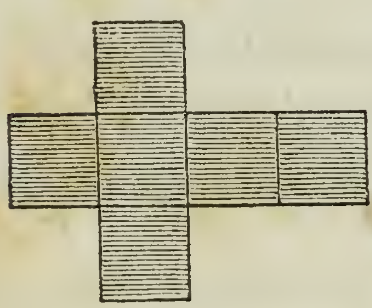


Pyramid unfolded.

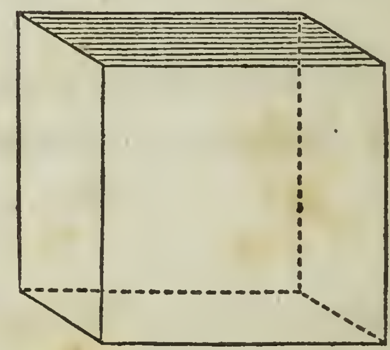


Pyramid.

2. The *hexaedron*, or cube, is a solid bounded by six equal squares.

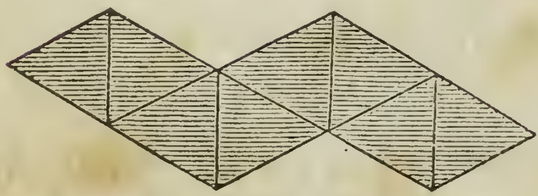


Cube unfolded.

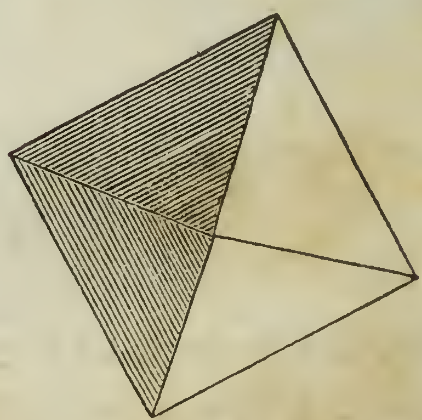


Cube.

3. The *octaedron*, is a solid bounded by eight equal triangles.

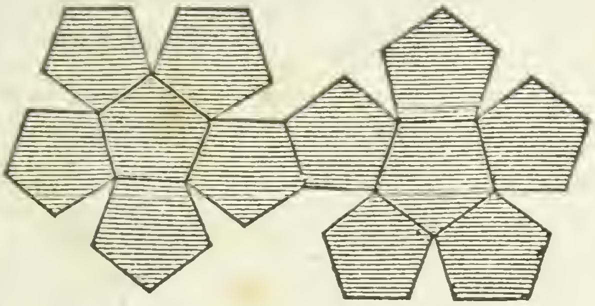


Octaedron unfolded.

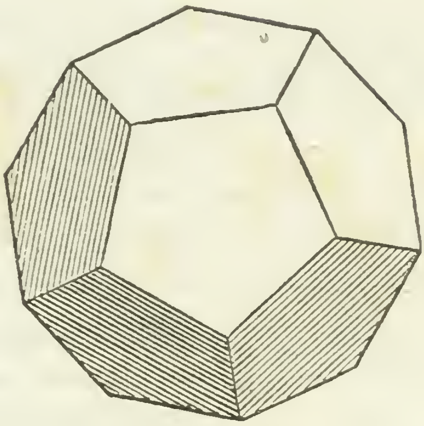


Octaedron.

4. The *dodecaedron* is a solid bounded by twelve equal pentagons.

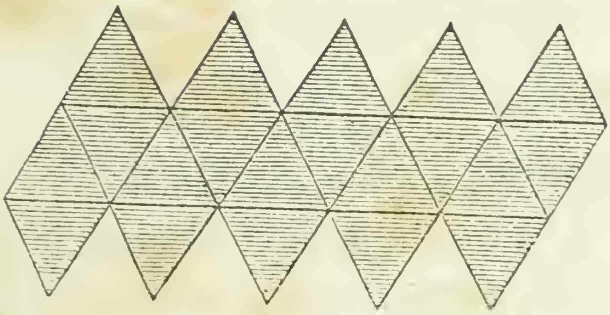


Dodecaedron unfolded.

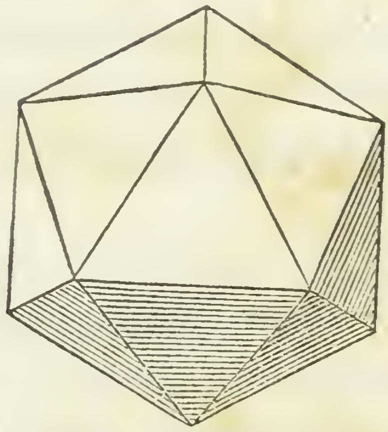


Dodecaedron.

5. The *icosaedron* is a solid bounded by twenty equal triangles.



Icosaedron unfolded.



Icosaedron.

6. The regular solids may easily be made of pasteboard.

Draw the figures of the unfolded regular solids accurately on pasteboard, and then cut through the bounding lines: this will give figures of pasteboard similar to the diagrams. Then, cut the other lines half through the pasteboard; after which, fold up the parts, and glue them together, and you will form the bodies which have been described.

The following table shows the surface and solidity of each of the regular solids, when the linear edge is unity.

No. of sides.	Names.	Surfaces.	Solidities.
4	Tetraedon	1.73205	0.11785
6	Hexaedron	6.00000	1.00000
8	Octaedron	3.46410	0.47140
12	Dodecaedron	20.64578	7.66312
20	Icosaedron	8.66025	2.18169

69. *How will you find the surface of a regular solid, when the length of the linear edge is given?*

Multiply the square of the linear edge by the tabular number in the column of surfaces, and the product will be the surface required.

EXAMPLES.

1. The linear edge of a tetraedron is 3: what is its surface?

The tabular area is 1.73205. Then,

$$\overline{3}^2 = 9; \text{ and } 1.73205 \times 9 = 15.58845 = \text{surface}.$$

2. The linear edge of an octaedron is 5: what is its surface?

The tabular area is 3.46410. Then,

$$\overline{5}^2 = 25; \text{ and } 3.46410 \times 25 = 86.6025 = \text{surface}.$$

3. The linear edge of an icosaedron is 6: what is its surface?

The tabular area is 8.66025. Then,

$$\overline{6}^2 = 36; \text{ and } 8.66025 \times 36 = 311.769 = \text{surface}.$$

70. *How do you find the solidity of a regular solid, when the length of a linear edge is known?*

Multiply the cube of the linear edge by the tabular number in the column of solidities, and the product will be the solidity required.

EXAMPLES.

1. What is the solidity of a regular tetraedron whose side is 6?

The tabular number in the column of solidities is 0.11785. Then,

$$\overline{6}^3 = 216; \text{ and } 0.11785 \times 216 = 25.4556.$$

2. What is the solidity of a regular octaedron whose linear edge is 8?

The tabular number in the column of solidities is 0.47140. Then,

$$\overline{8}^3 = 512; \text{ and } 0.47140 \times 512 = 241.35680 = \text{solidity}.$$

3. What is the solidity of a regular dodecaedron whose linear edge is 3?

The tabular number in the column of solidities is 7.66312. Then,

$$\overline{3}^3 = 27; \text{ and } 7.66312 \times 27 = 206.90424 = \text{solidity}.$$

4. What is the solidity of a regular icosaedron whose linear edge is 3?

The tabular number in the column of solidities is 2.18169. Then,

$$\overline{3}^3 = 27; \text{ and } 2.18169 \times 27 = 58.90563 = \text{solidity}.$$

BOOK VI.

ARTIFICERS' WORK.

SECTION I.

OF MEASURES.

1. *What is the* CARPENTER'S RULE *used for?*

The carpenter's rule, sometimes called the sliding rule, is used for the measurement of timber, and artificers' work. By it the dimensions are taken, and by means of certain scales, the superficial and solid contents may be computed.

2. *Describe the rule.*

The rule consists of two equal pieces of box wood, each one foot long, and connected together by a folding joint.

One face of the rule is divided into inches, half inches, quarter inches, eighths of inches, and sixteenths of inches When the rule is opened, the inches are numbered from 1 to 23, the last number 24, at the end, being omitted.

3. *How is the edge of the rule divided?*

The edge of the rule is divided decimally; that is, each foot is divided into ten equal parts, and each of those again into ten parts, so that the divisions on the edge of the scale are hundredths of a foot. The hundredths are numbered on each arm of the scale, from the right to the left.

4. *How are inches changed to the decimal of a foot?*

By means of the decimal divisions it is easy to convert inches into the decimal of a foot.

Thus, if we have 6 inches, we find its corresponding decimal on the edge of the rule to be 50 hundredths of a foot, or .50. Also 9 inches correspond to .75; 8 inches to .67 nearly, and 3 inches to .25.

5. *How are feet and inches multiplied by means of decimals?*

The multiplication of numbers is more easily made when the numbers are expressed decimally than when expressed in feet and inches.

Let us take an example. A board is 12 feet 6 inches long, and 2 feet 3 inches wide: how many square feet does it contain?

We see from the edge of the rule, that 6 inches correspond to .50, and 3 inches to .25. Hence, we have

By cross multiplication.

$$\begin{array}{rll} 12 & 6' & \\ 2 & 3' & \\ \hline 25 & & \\ 3 & 1' & 6'' \\ \hline 28 & 1' & 6'' \text{ content.} \end{array}$$

By decimals.

$$\begin{array}{r} 12.50 \\ 2.25 \\ \hline 6250 \\ 2500 \\ 2500 \\ \hline 28.1250 \text{ content.} \end{array}$$

6. *What are the objects of the scale marked* M *and* E?

Besides the scale of feet and inches, already referred to, there are, on the same side, two small scales, marked M and E; the first is numbered from 1 to 36, and the second from 1 to 26. The object of these scales is to change a square into what is called in carpentry *an eight square*, or regular octagon.

7. *Explain the use of the one marked* M.

Having formed the square which is to be changed to the octagon, find the middle of each side, and then the divisions of the scale marked M show the distances to be laid off on each side of the centre points, to give the angles of the octagon.

For example, if the side of the square is 6 inches, the distance to be laid off is found by extending the dividers from 1 to 6. If the side of the square is 12 inches, the distance to be taken reaches from 1 to 12; and so on for any distance from 1 to 36.

8. *Explain the use of the one marked* E.

The scale marked E is for the same object, only the distances are laid off from the angular points of the square instead of from the centre.

Thus, if we have a square whose side is 9 inches, and wish to change it into an octagon, take from the scale E the distance from 1 to 9, and mark it off from each angle of the square, on the sides: then join the points, and the figure so formed will be a regular octagon.

If the side of the square is 18 inches, the distance to be taken reaches from 1 to 18, and so for any distance between 1 and 26, the numbers on the scale pointing out the distances to be laid off when the side of the square is expressed in inches.

9. *What scales are on the opposite face of the rule, and how are they designated?*

Turning the rule directly over, there will be seen on one arm several scales of equal parts, which are similar to those described at page 36.

Fitting into the other arm is a small brass slide, of the same length as the rule. On the face of the slide are two

ranges of divisions, which are precisely alike. The upper is designated by the letter B, and is to be used with the scale on the rule directly above, which is designated by A; the lower divisions on the slide designated by the letter C, are to be used with the scale mark GIRT LINE, and also designated by the letter D. The scales B and C on the slide, are numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, and 1, from the left hand towards the right. From the middle point 1, the numbers go on 12, 2, 3, 4, 5, 6, 7, 8, 9, and 10. Now, the values which the parts of this scale may represent will depend on the value given to the unit at the left hand. If the unit at the left be called 1, then the 1 at the centre point will represent 10, and the 2 at the right 20, the 3 at the right 30, and the ten 100, and similarly for the intermediate divisions.

If the left-hand unit be called 10, then the 1 at the centre point will represent one hundred; the 2, two hundred; the 3, three hundred; and so on for the divisions to the right.

10. *How do you multiply two numbers together by the sliding rule?*

1st. Mark a number on the scale *A* to represent the multiplier.

2d. Then shove the slide until 1 on *B* stands opposite the multiplier on *A*.

3d. Then pass along on *B* until you find a number to represent the multiplicand; the number opposite on *A* will represent the product.

EXAMPLES.

1. Multiply 24 by 14.

Move the slide until 1 on *B* is opposite the second long mark at the right of 12, which is the division correspond-

ing to 14. Then pass along *B* to the fourth of the larger lines on the right of 2: this line marks the division on the scale *A*, which shows the product. Now we must remark that the unit on the product line is always ten times greater than the unit 1 at the left of the slide: and since in the example this unit was 10, it follows that the 3 on *A* will stand for 300, and each of the smaller divisions for 10; hence the product as shown by the scale is nearly 340, and by judging by the eye, we write it 336.

2. What is the product of 36 by 22?

Move the slide till 1 on *B* stands at 22 on *A*; then pass along on *B* to the 6th line between 3 and 4: the figures on *A* will then stand for hundreds, and the product will be pointed out a little to the right of the 9th line, between 7 and 8; or it will be 792.

3. A board is 16 feet 9 inches long, and 15 inches, or 1 foot and 3 inches wide: how many square feet does it contain?

First, 16 feet and 9 inches = 16.75 feet;
and 15 inches = 1.25 feet:

Place 1 on *B* at the line corresponding to 16, between 12 and 2 on *A*, and then move over three-fourths of the distance to the next long line to the right. Then looking along on *A*, one quarter of the distance between 1 and 2, we find the area of the board to be 21 feet, which is correct, very nearly.

4. The length of a board is 15 feet 8 inches, and the breadth 1 foot 6 inches: what is the superficial content?

15 feet 8 inches = 15.7 nearly.
1 foot 6 inches = 1.5 feet.

Then, place 1 on *B* at 15.7 on *A*, and 1 and a half on *B* will mark 23 and a half feet on *A*, which is the area very nearly.

11. *Explain the manner in which the girt-line is numbered.*

Below the slide, and on the same side with the scales already described, is a row of divisions marked GIRT-LINE, and numbered from 4 to 40. This line is also designated on the scale by the letter *D*. The object of this girt-line, which is to be used in conjunction with the sliding scale, is to find the solid content of timber.

12. *What is the quarter-girt, and how do you find it?*

The *quarter-girt*, as it is called in the language of mechanics, is one quarter the circumference of a stick of timber at its middle point. The quarter-girt, in squared timber, is found by taking a mean between the breadth and thickness.

Thus, if the breadth at the middle point is 4 feet 6 inches, and the thickness 3 feet 4 inches, we have

	ft.	*in.*	
	4	6	breadth
	3	4	depth
2)	7	10	
	3	11	quarter-girt.

and hence the quarter-girt is 3 feet 11 inches.

13. *When a stick of timber tapers regularly, how do you find the quarter-girt?*

If a stick of timber tapers regularly from one end to the other, the breadth and depth at the middle point may be found by taking the mean of the breadth and depth at the ends.

Thus, if the breadths at the ends are 1 foot 6 inches, and 1 foot 3 inches, the mean breadth will be 1 foot $4\frac{1}{2}$ inches. And, if the depths at the ends are 1 foot 3 inches,

and 1 foot, the mean depth or thickness will be 1 foot $1\frac{1}{2}$ inches; and the quarter-girt will be 1 foot 3 inches.

14. *How do you find, by the sliding rule, the solid content of a stick of timber, when the length and quarter-girt are known?*

1st. Reduce the length of the timber to feet and decimals of a foot, and the quarter-girt to inches.

2d. Note on scale *C* the number which expresses the length, and move the slide until this number falls at 12 on the girt-line.

3d. Pass along on the girt-line till you find the number which expresses the quarter-girt in inches, and the division which it marks on *C* will show the content of the timber in cubic feet.

EXAMPLES.

1. A piece of square timber is 3 feet 9 inches broad, 2 feet 7 inches thick, and 20 feet long: how many solid feet does it contain?

$$\begin{array}{rr} \textit{ft.} & \textit{in.} \\ 3 & 9 \\ 2 & 7 \\ \hline 2)6 & 4 \\ \hline 3 & 2 \\ \hline \end{array} \text{ quarter-girt} = 38 \text{ inches.}$$

Now, move the slide until 20 on *C* falls at 12 on the girt-line. If we take 1 on *C* at the left for 10, 2 will represent 20, which is placed opposite 12 on *D*. Then passing along the girt-line to division 38, we find the content on *C* to be a little over 200, say $200\frac{1}{2}$.

2. The length of a piece of timber is 18 feet 6 inches, the breadths at the greater and less ends are 1 foot 6 inches, and 1 foot 3 inches; and the thickness at the

greater and less end, 1 foot 3 inches and 1 foot: what is the solid content?

Here, the mean breadth is 1 foot $4\frac{1}{2}$ inches, the mean thickness 1 foot $1\frac{1}{2}$ inches, and the quarter-girt 1 foot 3 inches, or 15 inches.

Therefore, place 18.5 on *C*, at 12 on *D*, and pass along the girt-line to 15; the number on *C*, which is a little more than 28 and a half, will express the solid content.

TABLE FOR BOARD MEASURE.

15. *Explain the table rule for finding the content of boards.*

Besides the carpenter's rule with a slide, which we have just described, there is another folding rule without a slide, and on the face of which is a table to show the content of a board from 1 to 20 feet in length, and from 6 to 20 inches in width.

The upper line of the table shows the length of the board in feet, and the column at the left shows the width of the board in inches, from 6 to 20. For convenience, however, the table is often divided into two parts, which are placed by the side of each other.

EXAMPLES.

1. If your board is 6 inches wide, and 14 feet long, cast your eye along the top line till you come to 14; directly under you will find 7, which shows that the board contains 7 square feet.

2. If your board is 10 inches wide, and 16 feet long, cast your eye along the top line till you come to 16; then pass along down till you come to the line of 10: the number thus found is 13–4, which shows that the board contains 13 and 4 twelfths square feet.

The right-hand side of the table begins at 13 inches on the left-hand column.

3. What is the content of a board which is 13 feet long, and 19 inches wide?

Look along the upper line to 13; then descend to the line 19, where you will find the number 20–7, which shows that the board contains 20 and 7 twelfths square feet.

4. If your board is 17 inches wide, and 14 feet long, you will look under 14 till you come on to the line 17, where you will find the number 19–10; which shows that the board contains 19 and 10 twelfths square feet.

5. If you have a board 24 feet long, and 20 inches wide, first take the area for 20 feet in length, and then for 4 feet. Thus,

for 20 feet by 20 inches,	33	4	
for 4 feet by 20 inches,	6	8	
their sum gives	40	0	square feet.

Note.—Add as above for any different lengths or widths.

If your stuff is $1\frac{1}{2}$ inches thick, add half to it.

If 2 inches thick, you must double it.

The table on the four-fold Rule is not divided.

BOARD MEASURE.

16. *Explain the board measure, and the manner of using it.*

This is a measure two feet in length, of an octagonal form, that is, having eight faces.

On the line running round the measure, at the centre, we find the faces of the measure marked, in succession, by the figures 8, 9, 10, 11, 12, 13, 14, and 15; and we shall designate each face by the figure which thus marks it. We will likewise observe, that figures corresponding

to these, are also sometimes placed at one end of the measure.

Now, these figures at the centre of the measure correspond to the length of the board to be measured. Thus, if the board were 13 feet in length, place the thumb on the line 13 at the centre, and then apply the measure across the board, and the number on the face 13, which the width of the board marks, will express the number of square feet in the board. Thus, if the width of the board extended from 1 to 15, the board would contain 15 square feet.

If the board to be measured was 14 feet long, its content would be measured on face 14. If the board were 18 feet long, measure its width on face 8, and also on face 10, and take the sum for the true content of the board.

The Measures described above, are made by Jones & Co., of Hartford, Ct.

SECTION II.

OF TIMBER MEASURE.

1. *What methods have already been explained?*

The methods of finding both the superficial content of boards and the solid content of timber, by rules and scales, have already been given. We shall now give the more accurate methods by means of figures.

2. *How do you find the area of a board, or plank?*

Multiply the length by the breadth, and the product will be the content required.

3. *How do you find it when the board tapers?*

If the board is tapering, add the breadths of the two ends together, and take half the sum for a mean breadth, and multiply the result by the length.

4. *How may the examples be done?*

The examples may either be done by cross multiplication, or the inches may be reduced to the decimals of a foot, and the numbers then multiplied together.

EXAMPLES.

1. What is the area of a board whose length is 8 feet 6 inches, and breadth 1 foot 3 inches?

By cross multiplication.

$$\begin{array}{llll} \textit{ft.} & \textit{in.} & & \\ 8 & 6 & & \\ 1 & 3 & & \\ \hline 8 & 6' & & \\ 2 & 1 & 6'' & \\ \hline 10 & 7' & 6'' & \text{content.} \\ \hline \end{array}$$

By decimals.

$$\begin{array}{l} \textit{ft.}\ \textit{in.} \\ 8\ \ 6 = 8.\ 5\ \textit{ft.} \\ 1\ \ 3' = 1.25' \\ \hline \text{Product} = \underline{10.625}\ \textit{sq. ft.} \end{array}$$

2. What is the content of a board 12 feet 6 inches long, and 2 feet 3 inches broad?

Ans. 28 *ft.* 1′ 6″, or 28.125 *sq. ft.*

3. How many square feet in a board whose breadth at one end is 15 inches, at the other 17 inches, the length of the board being 6 feet?

Ans. 8.

4. How many square feet in a plank whose length is 20 feet, and mean breadth 3 feet 3 inches?

Ans. 65.

5. What is the value of a plank whose breadth at one end is 2 feet, and at the other 4 feet, the length of the plank being 12 feet, and the value per square foot 10 cents?

Ans. $3.60.

5. *Having given one dimension of a plank or board, how do you find the other dimension such, that the plank shall contain a given area?*

Divide the given area by the given dimension, and the quotient will be the other dimension.

EXAMPLES.

1. The length of a board is 16 feet; what must be its width that it may contain 12 square feet?

16 feet $= 192$ inches

12 square feet $= 144 \times 12 = 1728$ square inches.

Then, $1728 \div 192 = 9$ inches, the width of the board.

2. If a board is 6 inches broad, what length must be cut from it to make a square foot? *Ans.* 2 *ft.*

3. If a board is 8 inches wide, what length of it will make 4 square feet? *Ans.* 6 *ft.*

4. A board is 5 feet 3 inches long; what width will make 7 square feet? *Ans.* 1 *ft.* 4 *in.*

5. What is the content of a board whose length is 5 feet 7 inches, and breadth 1 foot 10 inches? *Ans.* 10 *ft.* 2′ 10″.

6. *How do you find the solid content of squared or four-sided timber which does not taper?*

Multiply the breadth by the depth, and then multiply the product by the length: the result will be the solid content.

EXAMPLES.

1. A squared piece of timber is 15 inches broad, 15 inches deep, and 18 feet long: how many solid feet does it contain? *Ans.* 28.125.

2. What is the solid content of a piece of timber whose breadth is 16 inches, depth 12 inches, and length 12 feet?

Ans. 16 *ft.*

3. The length of a piece of timber is 24.5 feet; its ends are equal squares, whose sides are each 1.04 feet: what is the solidity?

Ans. 26.4992 *solid ft.*

7. *How do you find the solidity of a squared piece of timber which tapers regularly?*

1st. Add together the breadths at the two ends, and also the depths.

2d. Multiply these sums together, and to the result add the products of the depth and breadth at each end.

3d. Multiply the last result by the length, and take one-sixth of the product, which will be the solidity.

EXAMPLES.

1. How many cubic feet in a piece of timber whose ends are rectangles, the length and breadth of the larger being 14 inches and 12 inches; and of the smaller, 6 and 4 inches, the length of the piece being $30\frac{1}{2}$ feet?

$$\begin{array}{rrr} 14 & 12 & 16 \times 20 = 320 \\ 6 & 4 & 14 \times 12 = 168 \\ \hline 20 & 16 & 6 \times 4 = 24 \\ & & \overline{512} \text{ square inches.} \end{array}$$

But, 512 square inches $= \frac{32}{9}$ square feet.
Then, $\frac{32}{9} \times 30\frac{1}{2} \times \frac{1}{6} = 18\frac{2}{27}$ solid feet.

2. How many solid inches in a mahogany log, the depth and breadth at one end being $81\frac{1}{2}$ inches and 55 inches, and of the other 41 and $29\frac{1}{2}$ inches, the length of the log being $47\frac{1}{4}$ inches?

Ans. 126340.59375.

3. How many cubic feet in a stick of timber whose larger end is 25 feet by 20, the smaller 15 feet by 10, and the length 12 feet?

Ans. 3700.

4. What is the number of cubic feet in a stick of hewn timber, whose ends are 30 inches by 27 and 24 inches by 18, the length being 24 feet?

Ans. 102.

5. The length of a piece of timber is 20.38 feet, and the ends are unequal squares: the side of the greater is $19\frac{1}{8}$ inches, and of the less $9\frac{7}{8}$ inches: what is the solid content?

Ans. 30.763 *cubic ft.*

6. The length of a piece of timber is 27.36 feet: at the greater end, the breadth is 1.78 feet and the thickness 1.23 feet; and at the less end, the breadth is 1.04 feet and the thickness 0.91 feet: what is its solidity?

Ans. 41.8179 *cubic ft.*

8. *How do you do when the timber does not taper regularly?*

If the timber does not taper regularly, measure parts of the stick, the same as if it had a regular taper, and take the sum of the parts for the entire solidity.

9. *Knowing the area of the end of a square piece of timber which does not taper, how do you find the length which must be cut off in order to obtain a given solidity?*

1st. Reduce the given solidity to cubic inches.

2d. Divide the number of cubic inches by the area of the end expressed in inches, and the quotient will be the length in inches.

EXAMPLES.

1. A piece of timber is 10 inches square: how much must be cut off to make a solid foot?

$10 \times 10 = 100$ square inches.

Then, $1728 \div 100 = 17.28$ inches.

2. A piece of timber is 20 inches broad and 10 inches deep: how much in length will make a solid foot?

Ans. $8\frac{16}{25}$ *in.*

3. A piece of timber is 9 inches broad and 6 inches deep: how much in length will make 3 solid feet?

Ans. 8 *ft.*

10. *How do you find the solidity of round or unsquared timber?*

1st. Take the girt or circumference, and then divide it by 5.

2d. Multiply the square of one-fifth of the girt by twice the length, and the product will be the solidity very nearly.

EXAMPLES.

1. A piece of round timber is $9\frac{3}{4}$ feet in length, and the girt is 13 feet: what is its solidity?

First, $13 \div 5 = 2.6$ the fifth of the girt.

Also, $\overline{2.6}^2 = 6.76$; and $9.75 \times 2 = 19.50$.

Again, $6.76 \times 19.5 = 131.82$ cubic feet, which is the required solidity.

2. The length of a tree is 24 feet, and the girt throughout 8 feet: what is the content?

Ans. 122.88 *cubic ft.*

3. Required the content of a piece of timber, its length being 9 feet 6 inches, and girt 14 feet.

Ans. 148.96 *cubic ft.*

11. *How do you do when the timber tapers?*

Gird the timber at as many points as may be necessary, and divide the sum of the girts by their number for the mean girt, of which take one-fifth, and proceed as before.

4. If a tree, girt 14 feet at the thicker end and 2 feet at the smaller end, be 24 feet in length, how many solid feet will it contain?

Ans. 122.88.

5. A tree girts at five different places as follows: in the first 9.43 feet; in the second 7.92 feet; in the third 6.15 feet; in the fourth 4.74 feet; and in the fifth 3.16 feet: now, if the length of the tree be 17.25 feet, what is its solidity?

Ans. 54.42499 *cubic ft.*

OF LOGS FOR SAWING.

12. *What is often necessary for lumber merchants?*

It is often necessary for lumber merchants to ascertain the number of feet of boards which can be cut from a given log; or, in other words, to find how many logs will be necessary to make a given amount of boards.

13. *What is a standard board?*

A standard board is one which is 12 inches wide, 1 inch thick, and 12 feet long: hence, a standard board is 1 inch thick and contains 12 square feet.

14. *What is a standard saw-log?*

A standard log is 12 feet long and 24 inches in diameter.

15. *How will you find the number of feet of boards which can be sawed from a standard log?*

If we saw off, say 2 inches, from each side, the log will be reduced to a square 20 inches on a side. Now, since a standard board is one inch in thickness, and since the saw cuts about one quarter of an inch each time it goes through, it follows that one-fourth of the log will be consumed by the saw. Hence we shall have

$$20 \times \frac{3}{4} = \text{the number of boards cut from the log.}$$

Now, if the width of a board in inches be divided by 12, and the quotient be multiplied by the length in feet, the product will be the number of square feet in the board.

Hence, $\frac{20}{12} \times$ length of the log in feet $=$ the square feet in each board. Therefore,

$$20 \times \frac{3}{4} \times \frac{20}{12} \times \text{length of log} = \text{the square feet in all}$$

the boards,

$$= 20 \times 10 \times \frac{3}{4} \times \frac{2}{12} \times \text{length of log} = 20 \times 10 \times \frac{1}{8} \times$$

length; and the same may be shown for a log of any length.

16. *What then is the rule for finding the number of feet of boards which can be cut from any log whatever?*

From the diameter of the log, in inches, subtract 4 for the slabs. Then multiply the remainder by half itself and the product by the length of the log, in feet, and divide the result by 8: the quotient will be the number of square feet.

EXAMPLES.

1. What is the number of feet of boards which can be cut from a standard log?

Diameter	24 inches
for slabs	4
remainder	20
half remainder	10
	200
length of log	12
	8)2400
	300 = the number of feet.

2. How many feet can be cut from a log 12 inches in diameter and 12 feet long? *Ans.* 48.

3. How many feet can be cut from a log 20 inches in diameter and 16 feet long? *Ans.* 256.

4. How many feet can be cut from a log 24 inches in diameter and 16 feet long? *Ans.* 400.

5. How many feet can be cut from a log 28 inches in diameter and 14 feet long? *Ans.* 504.

SECTION III.

BRICKLAYERS' WORK.

1. *In how many ways is artificers' work computed?*

Artificers' work, in general, is computed by three different measures, viz.:

1st. The linear measure, or, as it is called by mechanics, running measure.

2d. Superficial or square measure, in which the computation is made by the square foot, square yard, or by the square containing 100 square feet, or yards.

3d. By the cubic or solid measure, when it is estimated by the cubic foot, or the cubic yard. The work, however, is often estimated in square measure, and the materials for construction in cubic measure.

2. *What proportion do the dimensions of a brick bear to each other?*

The dimensions of a brick generally bear the following proportions to each other, viz.:

Length = twice the width, and
Width = twice the thickness, and

hence, the length is equal to four times the thickness.

3. *What are the common dimensions of a brick? How many cubic inches does it contain?*

The common length of a brick is 8 inches, in which case the width is 4 inches, and the thickness 2 inches. A brick of this size contains

$8 \times 4 \times 2 = 64$ cubic inches; and since a cubic foot contains 1728 cubic inches, we have

$1728 \div 64 = 27$ the number of bricks in a cubic foot.

4. *If a brick is 9 inches long, what will be its width and what its content?*

If the brick is 9 inches long, then the width is $4\frac{1}{2}$ inches, and the thickness $2\frac{1}{4}$; and then each brick will contain

$9 \times 4\frac{1}{2} \times 2\frac{1}{4} = 91\frac{1}{8}$ cubic inches in each brick; and

$1728 \div 91\frac{1}{8} = 19$ nearly, the number of bricks in a cubic foot. In the examples which follow, we shall suppose the brick to be 8 inches long.

5. *How do you find the number of bricks required to build a wall of given dimensions?*

1st. Find the content of the wall in cubic feet.

2d. Multiply the number of cubic feet by the number of bricks in a cubic foot, and the result will be the number of bricks required.

EXAMPLES.

1. How many bricks, of 8 inches in length, will be required to build a wall 30 feet long, a brick and a half thick, and 15 feet in height?

Ans. 12150.

2. How many bricks, of the usual size, will be required to build a wall 50 feet long, 2 bricks thick, and 36 feet in height?

Ans. 64800.

6. *What allowance is made for the thickness of the mortar?*

The thickness of mortar between the courses is nearly a quarter of an inch, so that four courses will give nearly 1 inch in height. The mortar, therefore, adds nearly one-eighth to the height; but as one-eighth is rather too large an allowance, we need not consider the mortar which goes to increase the length of the wall.

3. How many bricks would be required in the first and second examples, if we make the proper allowance for mortar?

Ans. $\begin{cases} \text{1st. } 10631\frac{1}{4}. \\ \text{2d. } 56700. \end{cases}$

7. *How do bricklayers generally estimate their work?*

Bricklayers generally estimate their work at so much per thousand bricks. To find the value of things estimated by the thousand, see Arithmetic, page 192.

4. What is the cost of a wall 60 feet long, 20 feet high, and two and a half bricks thick, at $7.50 per thousand, which price we suppose to include the cost of the mortar?

If we suppose the mortar to occupy a space equal to one-eighth the height of the wall, we must find the quantity of bricks under the supposition that the wall was $17\frac{1}{2}$ feet in height.

Ans. \$354.37$\frac{1}{2}$.

8. *In estimating the bricks for a house, what allowances are made?*

In estimating the bricks for a house, allowance must be made for the windows and doors.

OF CISTERNS.

9. *What are cisterns?*

Cisterns are large reservoirs constructed to hold water, and to be permanent, should be made either of brick or masonry.

It frequently occurs that they are to be so constructed as to hold given quantities of water, and it then becomes a useful and practical problem to calculate their exact dimensions.

10. *How many cubic inches in a hogshead?*

It was remarked in Arithmetic, page 104, that a hogshead contains 63 gallons, and that a gallon contains 231 cubic inches. Hence, $231 \times 63 = 14553$, the number of cubic inches in a hogshead.

11. *How do you find the number of hogsheads which a cistern of given dimensions will contain?*

1st. Find the solid content of the cistern in cubic inches.

2d. Divide the content so found by 14553, and the quotient will be the number of hogsheads.

EXAMPLE.

The diameter of a cistern is 6 feet 6 inches, and height 10 feet: how many hogsheads does it contain?

The dimensions reduced to inches are 78 and 120. To find the solid content, see page 162. Then, the content in cubic inches, which is 573404.832, gives

$$573404.832 \div 14553 = 39.40 \text{ hogsheads, nearly.}$$

12. *If the height of a cistern be given, how do you find the diameter, so that the cistern shall contain a given number of hogsheads?*

1st. Reduce the height of the cistern to inches, and the content to cubic inches.

2d. Multiply the height by the decimal .7854.

3d. Divide the content by the last result, and extract the square root of the quotient, which will be the diameter of the cistern in inches.

EXAMPLE.

The height of a cistern is 10 feet: what must be its diameter, that it may contain 40 hogsheads?

Ans. 78.6 *in. nearly.*

13. *If the diameter of a cistern be given, how do you find the height, so that the cistern shall contain a given number of hogsheads?*

1st. Reduce the content to cubic inches.

2d. Reduce the diameter to inches, and then multiply its square by the decimal .7854.

3d. Divide the content by the last result, and the quotient will be the height in inches.

EXAMPLE.

The diameter of a cistern is 8 feet: what must be its height that it may contain 150 hogsheads?

Ans. 25 *ft.* 1 *in. nearly.*

SECTION IV.

MASONS' WORK.

1. *What belongs to* MASONRY, *and what measures are used?*

All sorts of stone work. The measure made use of is either superficial or solid.

Walls, columns, blocks of stone or marble, are measured by the cubic foot; and pavements, slabs, chimney-pieces, &c., are measured by the square or superficial foot. Cubic or solid measure is always used for the materials, and the square measure is sometimes used for the workmanship.

EXAMPLES.

1. Required the solid content of a wall 53 feet 6 inches long, 12 feet 3 inches high, and 2 feet thick.

Ans. $1310\frac{3}{4}$ *ft.*

2. What is the solid content of a wall, the length of which is 24 feet 3 inches, height 10 feet 9 inches, and thickness 2 feet?

Ans. 521.375 *ft.*

3. In a chimney-piece we find the following dimensions:

Length of the mantel and slab,	4	feet	2	inches.
Breadth of both together,	3	"	2	"
Length of each jamb,	4	"	4	"
Breadth of both,	1	"	9	"

Required the superficial content.

Ans. 21 *ft.* 10′.

SECTION V.

CARPENTERS' AND JOINERS' WORK.

1. *In what does carpenters' and joiners' work consist?*

Carpenters' and joiners' work is that of flooring, roofing, &c., and is generally measured by the square of 100 square feet.

2. *When is a roof said to have a true pitch?*

In carpentry, a roof is said to have a *true pitch* when the length of the rafters is three-fourths the breadth of the building. The rafters then are nearly at right angles. It is therefore customary to take once and a half times the area of the flat of the building for the area of the roof.

EXAMPLES.

1. How many squares, of 100 square feet each, in a floor 48 feet 6 inches long, and 24 feet 3 inches broad?

Ans. 11 and $76\frac{1}{8}$ *sq. ft.*

2. A floor is 36 feet 3 inches long, and 16 feet 6 inches broad: how many squares does it contain?

Ans. 5 and $98\frac{1}{8}$ *sq. ft.*

3. How many squares are there in a partition 91 feet 9 inches long, and 11 feet 3 inches high?

Ans. 10 and 32 *sq. ft.*

4. If a house measure within the walls 52 feet 8 inches in length, and 30 feet 6 inches in breadth, and the roof be of the true pitch, what will the roofing cost at $1.40 per square?

Ans. $33.733.

OF BINS FOR GRAIN.

3. *What is a bin?*

It is a wooden box used by farmers for the storage of their grain.

4. *Of what form are bins generally made?*

Their bottoms or bases are generally rectangles, and horizontal, and their sides vertical.

5. *How many cubic feet are there in a bushel?*

Since a bushel contains 2150.4 cubic inches, (see Arithmetic, page 106,) and a cubic foot 1728 inches, it follows that a bushel contains one and a quarter cubic feet, nearly.

6. *Having any number of bushels, how then will you find the corresponding number of cubic feet?*

Increase the number of bushels one-fourth itself, and the result will be the number of cubic feet.

EXAMPLES.

1. A bin contains 372 bushels; how many cubic feet does it contain?

$372 \div 4 = 93$; hence, $372 + 93 = 465$ cubic feet.

2. In a bin containing 400 bushels, how many cubic feet? *Ans.* 500.

7. *How will you find the number of bushels which a bin of a given size will hold?*

Find the content of the bin in cubic feet; then diminish the content by one-fifth, and the result will be the content in bushels.

3. A bin is 8 feet long, 4 feet wide, and 5 feet high how many bushels will it hold?

$$8 \times 4 \times 5 = 160$$

then, $160 \div 5 = 32 : 160 - 32 = 128$ bushels = capacity of bin.

4. How many bushels will a bin contain which is 7 feet long, 3 feet wide, and 6 feet in height?

Ans. 100.8 *bush.*

8. *How will you find the dimensions of a bin which shall contain a given number of bushels?*

Increase the number of bushels one-fourth itself, and the result will show the number of cubic feet which the bin will contain. Then, when two dimensions of the bin are known, divide the last result by their product, and the quotient will be the other dimension.

5. What must be the height of a bin that will contain 600 bushels, its length being 8 feet and breadth 4?
$600 \div 4 = 150$; hence, $600 + 150 = 750$ = the cubic feet; and $8 \times 4 = 32$, the product of the given dimensions. Then, $750 \div 32 = 23.44$ feet, the height of the bin.

6. What must be the width of a bin that shall contain 900 bushels, the height being 12 and the length 10 feet?
$900 \div 4 = 225$; hence, $900 + 225 = 1125$ = the cubic feet; and $12 \times 10 = 120$, the product of the given dimensions. Then, $1125 \div 120 = 9.375$ feet, the width of the bin.

7. The length of a bin is 4 feet, its breadth 5 feet 6 inches: what must be its height that it may contain 136 bushels?

Ans. 7 *ft.* 8 *in.* +

8. The depth of a bin is 6 feet 2 inches, the breadth 4 feet 8 inches: what must be the length that it may contain 200 bushels?

Ans. 104 *in.* +

SECTION VI.

SLATERS' AND TILERS' WORK.

1. *How is the content of a roof found?*

In this work, the content of the roof is found by multiplying the length of the ridge by the girt from eaves to eaves. Allowances, however, must be made for the double rows of slate at the bottom.

EXAMPLES.

1. The length of a slated roof is 45 feet 9 inches, and its girt 34 feet 3 inches: what is its content?

Ans. 1566.9375 *sq. ft.*

2. What will tne tiling of a barn cost, at $3.40 per square of 100 feet, the length being 43 feet 10 inches, and breadth 27 feet 5 inches, on the flat, the eave-board projecting 16 inches on each side, and the roof being of the true pitch?

Ans. $65.26.

SECTION VII.

PLASTERERS' WORK.

1. *How many kinds of plasterers' work are there, and how are they measured?*

Plasterers' work is of two kinds, viz.: ceiling, which is plastering on laths; and rendering, which is plastering on walls. These are measured separately.

The contents are estimated either by the square foot, the square yard, or by the square of 100 feet.

Inriched mouldings, &c., are rated by the running or lineal measure.

In estimating plastering, deductions are made for chimneys, doors, windows, &c.

EXAMPLES.

1. How many square yards are contained in a ceiling 43 feet 3 inches long, and 25 feet 6 inches broad?

Ans. $122\frac{1}{2}$ *nearly.*

2. What is the cost of ceiling a room 21 feet 8 inches, by 14 feet 10 inches, at 18 cents per square yard?

Ans. $\$6.42\frac{1}{9}$.

3. The length of a room is 14 feet 5 inches, breadth 13 feet 2 inches, and height to the under side of the cornice 9 feet 3 inches. The cornice girts $8\frac{1}{2}$ inches, and projects 5 inches from the wall on the upper part next the ceiling, deducting only for one door 7 feet by 4: what will be the amount of the plastering?

$$Ans. \begin{cases} 53 \; yds. \; 5 \; ft. \; 3' \; 6'' \; of \; rendering. \\ 18 \; yds. \; 5 \; ft. \; 6' \; 4'' \; of \; ceiling. \\ 37 \; ft. \; 10' \; 9'' \; of \; cornice. \end{cases}$$

How is the area of the cornice found in the above examples?

The mean length of the cornice, both in the length and breadth of the house, is found by taking the middle line of the cornice. Now, since the cornice projects 5 inches at the ceiling, it will project $2\frac{1}{2}$ inches at the middle line; and therefore, the length of the middle line along the length of the room will be 14 feet, and across the room, 12 feet 9 inches. Then multiply the double of each of these numbers by the girth, which is $8\frac{1}{2}$ inches, and the sum of the products will be the area of the cornice.

SECTION VIII.

PAINTERS' WORK.

How is painters' work computed, and what allowances are made?

Painters' work is computed in square yards. Every part is measured where the color lies, and the measuring line is carried into all the mouldings and cornices.

Windows are generally done at so much a piece. It is usual to allow double measure for carved mouldings, &c.

EXAMPLES.

1. How many yards of painting in a room which is 65 feet 6 inches in perimeter, and 12 feet 4 inches in height?
Ans. $89\frac{4}{5}\frac{1}{4}$ *sq. yds.*

2. The length of a room is 20 feet, its breadth 14 feet 6 inches, and height 10 feet 4 inches: how many yards of painting are in it, deducting a fire-place of 4 feet by 4 feet 4 inches, and two windows, each 6 feet by 3 feet 2 inches? *Ans.* $73\frac{2}{27}$ *sq. yds.*

SECTION IX.

PAVERS' WORK.

How is pavers' work estimated?

Pavers' work is done by the square yard, and the content is found by multiplying the length and breadth together.

EXAMPLES.

1. What is the cost of paving a side-walk, the length

of which is 35 feet 4 inches, and breadth 8 feet 3 inches, at 54 cents per square yard? *Ans.* \$17.48 9.

2. What will be the cost of paving a rectangular courtyard, whose length is 63 feet, and breadth 45 feet, at 2*s.* 6*d.* per square yard; there being, however, a walk running lengthwise 5 feet 3 inches broad, which is to be flagged with stone costing 3 shillings per square yard?

Ans. £40 5*s.* 10½*d.*

SECTION X.

PLUMBERS' WORK.

1. Plumbers' work is rated at so much a pound, or else by the hundred weight. Sheet lead, used for gutters, &c., weighs from 6 to 12 lbs. per square foot. Leaden pipes vary in weight according to the diameter of their bore and thickness.

The following table shows the weight of a square foot of sheet lead, according to its thickness; and the common weight of a yard of leaden pipe, according to the diameter of the bore.

Thickness of lead.	Pounds to a square foot.	Bore of leaden pipes.	Pounds per yard.
Inch. $\frac{1}{10}$	5.899	Inch. $0\frac{3}{4}$	10
$\frac{1}{9}$	6.554	1	12
$\frac{1}{8}$	7.373	$1\frac{1}{4}$	16
$\frac{1}{7}$	8.427	$1\frac{1}{2}$	18
$\frac{1}{6}$	9.831	$1\frac{3}{4}$	21
$\frac{1}{5}$	11.797	2	24

EXAMPLES.

1. What weight of lead of $\frac{1}{10}$ of an inch in thickness, will cover a flat 15 feet 6 inches long, and 10 feet 3 inches broad, estimating the weight at 6 lbs. per square foot?

Ans. 8 *cwt.* 2 *qr.* $1\frac{1}{4}$*lb.*

2. What will be the cost of 130 yards of leaden pipe of an inch and a half bore, at 8 cents per pound, supposing each yard to weigh 18 lbs.?

Ans. $187.20.

3. The lead used for a gutter is 12 feet 5 inches long and 1 foot 3 inches broad: what is its weight, supposing it to be $\frac{1}{9}$ of an inch in thickness?

Ans. 101 *lbs.* 12 *oz.* 13.6 *dr.*

4. What is the weight of 96 yards of leaden pipe, of an inch and a quarter bore?

Ans. 13 *cwt.* 2 *qr.* 24 *lbs.*

5. What will be the cost of a sheet of lead 16 feet 6 inches long and 10 feet 4 inches broad, at 5 cents per pound; the lead being $\frac{1}{6}$ of an inch in thickness?

Ans. $83.81.

BOOK VII.

INTRODUCTION TO MECHANICS.

SECTION I.

OF MATTER AND BODIES.

1. *What is matter?*

MATTER is a general name for every thing which has substance, and is always capable of being increased or diminished. Whatever we can touch, taste, smell, or see, is matter.

2. *What is a body?*

A BODY is any portion of matter.

3. *What is space? How many dimensions has it?*

SPACE is mere extension, in which all bodies are situated. Thus, when a body has a certain place, it is said to occupy that portion of space which it fills. Space has three dimensions—length, breadth, and thickness.

4. *What are the properties common to all bodies?*

The properties which belong to all bodies are: *impenetrability*, *extension*, *figure*, *divisibility*, *inertia*, and *attraction*.

5. *What is impenetrability?*

IMPENETRABILITY is the property, in virtue of which a

body must fill a certain space, and which no other body can occupy at the same time. Thus, if you fill a vessel full of water, and then plunge in your hand, or a stick, some of the water will be forced over the top of the vessel. Your hand or the stick removes the water, and does not occupy the space until after the water is displaced.

6. *What is extension?*

Since a body occupies space, it must, like any portion of space, have the dimensions of length, breadth, and thickness. These are called the *dimensions of extension*, and vary in different bodies. The length, breadth, and depth of a house are very different from those of an inkstand.

7. *How are length and breadth measured? How do you measure height and depth?*

Length and breadth are generally measured in a horizontal direction. Height and depth are the same dimension: height is measured upward, and depth downward. Thus, we say a mountain is 400 feet high, and a river 50 feet deep.

8. *What is figure?*

FIGURE is merely the limit of extension. Figure is also called *form* or *shape*.

9. *When is a body said to be regular? when irregular?*

If all the parts of a body are arranged in the same way, about a line or a centre, the body is said to be *regular* or *symmetrical;* and when the parts are not so arranged, the body is said to be irregular. Nature has given regular forms to nearly all her productions.

10. *What is divisibility?*

DIVISIBILITY denotes the susceptibility of matter to be

continually divided. That is, a portion of matter may be divided, and each part again divided, and each of the parts divided again, and so on, continually, without ever arriving at a portion which will be absolutely nothing.

Suppose, for instance, you take a portion of matter, say one pound or one ounce, and divide it into two equal parts, and then divide each part again into two equal parts, and so on continually. Now, all the parts will continually grow smaller and smaller, but no one of them will ever become equal to nothing, since the half of a thing must always have some value.

11. *What is inertia?*

INERTIA is the resistance which matter makes to a change of state. Bodies are not only incapable of changing their actual state, whether it be that of motion or rest, but they seem endowed with the power of resisting such a change. This property is called *inertia.*

12. *If a body is at rest, will it remain so? If in motion, will it continue so?*

If a body is at rest it will remain so, unless something be applied from without to move it; and if it be moving, it will continue to move, unless something stops it.

13. *What are atoms?*

The smallest parts into which we can suppose a body divided, are called *particles* or *atoms.*

14. *Do these atoms adhere to each other?*

They do, and form masses or bodies.

15. *What is the force called which unites them?*

It is called the *attraction of cohesion.* Without this power solid bodies would crumble to pieces and fall to atoms.

16. *In what kind of bodies does this attraction exist?*

In all bodies, fluid as well as solid. It is the attraction of cohesion which holds a drop of water in suspension at the end of the finger, and causes it to take a spherical form.

The attraction of cohesion is stronger in some substances than in others. Those in which it is the weakest are easily broken, or the attraction is easily overcome; while those in which it is greater, are proportionably stronger.

17. *What is the difference between the attraction of cohesion and the attraction of gravitation?*

The attraction of cohesion unites the particles of matter, and these by their aggregation form masses or bodies. The *attraction of gravitation* is the force by which masses of matter tend to come together. The attraction of cohesion acts only between particles of matter which are very near each other, while the attraction of gravitation acts between bodies widely separated.

18. *Is the attraction between bodies mutual?*

The attraction between two bodies is mutual; that is, each body attracts the other just as much, and no more, than it is attracted by it. But if the bodies are left free, the smaller will move towards the larger; for, as they are urged together by *equal forces*, the smaller will obey the force faster than the larger. Thus, the earth being larger than any body near its surface, forces all bodies towards it, and they immediately fall unless the attraction of gravitation is counteracted.

It should, however, be borne in mind that every body attracts the earth just as strongly as the earth attracts the body; and the body moves towards the earth, only because the earth is larger, and therefore not as rapidly moved by their mutual attraction.

19. *What is weight?*

Weight is the force which is necessary to overcome the attraction of gravitation. Thus, if we have two bodies, and one has twice as much tendency to descend towards the earth as the other, it will require just twice as much force to support it, and hence we say that it is twice as heavy.

SECTION II.

LAWS OF MOTION, AND CENTRE OF GRAVITY.

1. *What is motion?*

Motion is a change of place. Thus, a body is said to be in motion when it is continually changing its place.

2. *Can a body put in motion stop itself?*

It has been observed in Art. 11, that bodies are indifferent to rest or motion. Hence, a body cannot put itself in motion, or stop itself after it has begun to move.

3. *What is force or power?*

That which puts a body in motion, or which changes its motion after it has begun to move, is called *force* or *power*. Thus, the stroke of the hammer is the force which drives the nail, the effort of the horse the force which moves the carriage, and the attraction of gravitation the force which draws bodies to the earth.

4. *What is velocity?*

The rate at which a body moves, or the rapidity of its motion, is estimated by the space which it passes over in

a given portion of time, and this rate is called its *velocity.* Thus, if in one minute of time a body passes over 200 feet, its velocity is said to be 200 feet per minute; and if another body, in the same time, passes over 400 feet, its velocity is said to be 400 feet per minute, or double that of the first.

5. *What is uniform velocity?*

When a body moves over equal distances in equal times, its velocity is said to be *uniform.* Thus, if a body move at the rate of 30 feet a second, it has a uniform velocity, for it always passes over an equal space in an equal time.

6. *What is uniformly-accelerated velocity?*

Bodies which receive uniform accelerations of velocity, that is, equal accelerations in equal times, are said to have motions *uniformly accelerated.*

7. *How will a body fall by the attraction of gravitation?*

If a body fall freely towards the earth, by the attraction of gravitation, it will descend in a line perpendicular to its surface.

8. *How far will it fall in the first second, and how far in each succeeding second? What kind of velocity will it have?*

In the first second it will fall through 16 feet; in the second second, having the velocity already acquired, and being still acted on by the force of gravity, it will descend through 32 feet; in the third second it will descend through 48 feet; in the fourth second through 64 feet, and so on, adding to its velocity in every additional second. This is a motion uniformly accelerated, for the velocity is equally increased in each second of time.

9. *What is momentum?*

Momentum is the force with which a body in motion would strike against another body. If a body of a given weight, say 10 pounds, were moving at the rate of 30 feet per second, and another body of the same weight were to move twice as fast, the last would have double the momentum of the first.

10. *On what does the momentum of a body depend?*

When the bodies are of a given weight, the momentum will depend on the velocity. But if two unequal bodies move with the same velocity, their momentum will depend upon their weight. Hence, the momentum of a body will depend on its weight and velocity; that is, it will be equal to the weight multiplied by the velocity.

If the weight of a body be represented by 5, and its velocity by 6, its momentum will be $5 \times 6 = 30$.

If the weight of a body be represented by 8, and its velocity by 2, its momentum will be represented by

$$16 \times 2 = 32.$$

11. *What are action and reaction, and how do they compare with each other?*

When a body in motion strikes against another body, it meets with resistance. The force of the moving body is called *action*, and the resistance offered by the body struck is called *reaction;* and it is a general principle, that action and reaction are equal. Thus, if you strike a nail with a hammer, the action of the hammer against the nail is just equal to the reaction of the nail against the hammer. Also, if a body fall to the earth, by the attraction of gravitation, the action of the body when it strikes the earth is just equal to the reaction of the earth against the body.

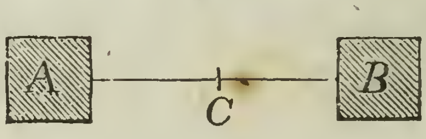
12. *What is the centre of gravity?*

The centre of gravity is that point of a body about which all the parts will exactly balance each other. Hence, if the centre of gravity be supported, the body will not fall, for all the parts will balance each other about the centre of gravity.

13. *Is the centre of gravity changed by changing the position of a body?*

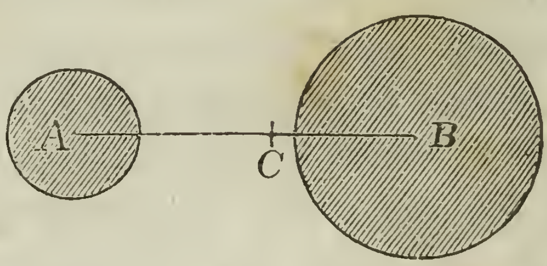
The centre of gravity of a body is not changed by changing the position of the body. Thus, if a body be suspended by a cord, attached at its centre of gravity, it will remain balanced, in every position of the body.

14. *If two equal bodies are joined by a bar, where will be the centre of gravity?*



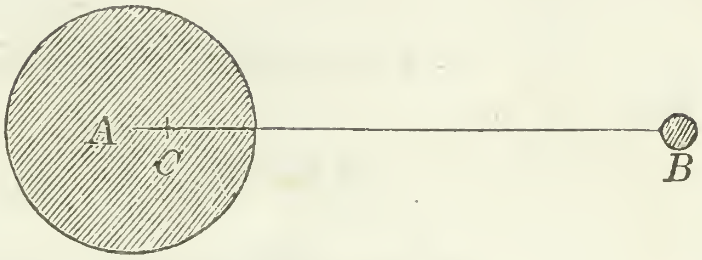
If we have two equal bodies A and B, connected together by a bar AB, the centre of gravity will be at C, the middle point of AB, and about this point the bodies will exactly balance each other.

15. *If the bodies are unequal, where will it be found?*



If we have two unequal bodies A and B, the centre of gravity C will be nearer the larger body than the smaller, and just as much nearer as the larger body exceeds the smaller. Thus, if B is three times greater than A, then BC will be one-third of AC.

16. *If one body is very large in comparison with the other, what will take place?*

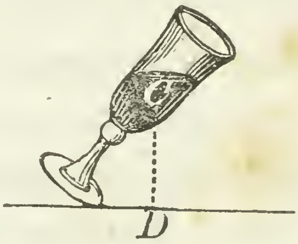


If one of the bodies is very large in comparison with the other, the centre of gravity may fall within the larger body. Thus, the centre of gravity of the bodies *A* and *B* falls at *C*.

17. *What is the line of direction of the centre of gravity?*

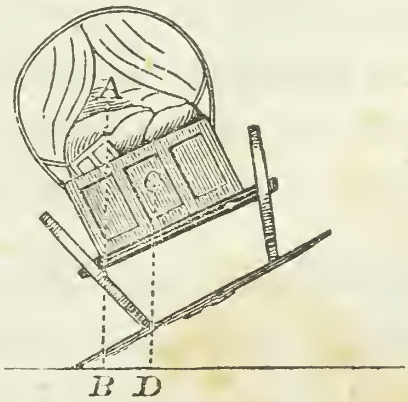
The vertical line drawn through the centre of gravity, is called the *line of direction* of the centre of gravity.

18. *If this line is supported, will the body fall?*



If the line of direction of the centre of gravity falls within the base on which the body stands, the body will be supported; but if the line falls without the base, the body will fall. Thus, if in a wine glass, the centre of gravity be at *C*, the glass will fall the moment the line *CD* falls without the base.

19. *If the line of direction of the centre of gravity falls near the base, is the body likely to fall? Give the illustration.*



Let us suppose a cart on inclined ground to be loaded with stone, so that the centre of gravity of the mass shall fall at *C*. In this position the line of direction *CD* falls within the base, and the cart will stand. But if the cart be loaded with hay, so as to bring

the centre of gravity at *A*, the line of direction *AB* will fall without the base, and the cart will be upset.

SECTION III.

OF THE MECHANICAL POWERS.

1. *How many mechanical powers are there, and what are they?*

There are six simple machines, which are called *Mechanical powers.* They are the *Lever*, the *Pulley*, the *Wheel* and *Axle*, the *Inclined Plane*, the *Wedge*, and the *Screw.*

2. *What four things must be considered, in order to understand the power of a machine?*

1st. The power or force which acts. This consists in the effort of men or horses, of weights, springs, steam, &c.

2d. The resistance which is to be overcome by the power. This generally is a weight to be moved.

3d. We are to consider the centre of motion, or *fulcrum*, which means a prop. The prop or fulcrum is the point about which all the parts of the machine move.

4th. We are to consider the respective velocities of the power and resistance.

3. *When is a machine said to be in equilibrium?*

A machine is said to be in equilibrium when the resistance exactly balances the power, in which case all the parts of the machine are at rest.

4. *What is a Lever?*

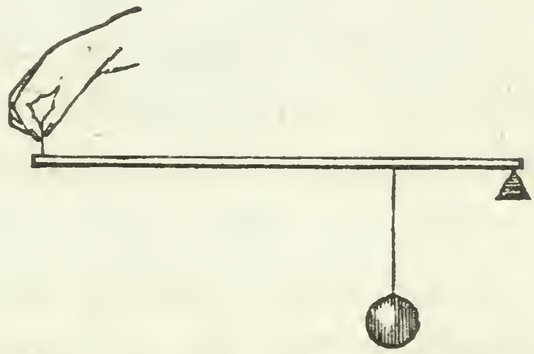
The lever is a straight bar of wood or metal, which moves around a fixed point called the fulcrum.

5. *How many kinds of levers are there?*

There are three kinds of levers:

1st. When the fulcrum is between the weight and the power.

2d. When the weight is between the power and the fulcrum.



3d. When the power is between the fulcrum and the weight.

6. *What are the arms of a lever?*

The parts of the lever, from the fulcrum to the weight and power, are called the *arms* of the lever.

7. *When is an equilibrium produced in the lever?*

An equilibrium is produced in all the levers when the weight, multiplied by its distance from the fulcrum, is equal to the product of the power multiplied by its distance from the fulcrum. That is,

The weight is to the power, as the distance from the power to the fulcrum, to distance from the weight to the fulcrum.

EXAMPLES.

1. In a lever of the first kind the fulcrum is placed at the middle point: what power will be necessary to balance a weight of 40 pounds?

2. In a lever of the second kind, the weight is placed at the middle point: what power will be necessary to sustain a weight of 50 lbs.?

3. In a lever of the third kind, the power is placed at the middle point: what power will be necessary to sustain a weight of 25 lbs.?

4. A lever of the first kind is 8 feet long, and a weight of 60 lbs. is at a distance of 2 feet from the fulcrum: what power will be necessary to balance it?

Ans. 20 *lbs.*

5. In a lever of the first kind, that is 6 feet long, a weight of 200 lbs. is placed at 1 foot from the fulcrum: what power will balance it?

Ans. 40 *lbs.*

6. In a lever of the first kind, like the common steelyard, the distance from the weight to the fulcrum is one inch: at what distance from the fulcrum must the poise of 1 lb. be placed, to balance a weight of 1 lb.? A weight of $1\frac{1}{2}$ lbs.? Of 2 lbs.? Of 4 lbs.?

7. In a lever of the third kind, the distance from the fulcrum to the power is 5 feet, and from the fulcrum to the weight 8 feet: what power is necessary to sustain a weight of 40 lbs.?

Ans. 64 *lbs.*

8. In a lever of the third kind, the distance from the fulcrum to the weight is 12 feet, and to the power 8 feet: what power will be necessary to sustain a weight of 100 lbs.?

Ans. 150 *lbs.*

8. *How are the equilibriums of levers affected by considering their weight?*

In levers of the first kind, the weight of the lever gener-

ally adds to the power, but in the second and third kinds, the weight goes to diminish the effect of the power.

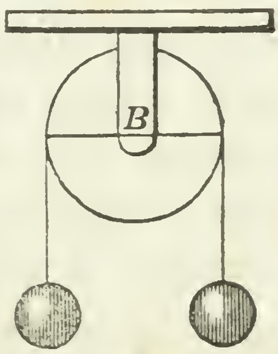
9. *What has been stated in the previous examples? What is necessary that the machine may move?*

In the previous examples, we have stated the circumstances under which the power will exactly sustain the weight. In order that the power may overcome the resistance, it must of course be somewhat increased. The lever is a very important mechanical power, being much used, and entering indeed into all the other machines.

OF THE PULLEY.

10. *What is a pulley?*

The pulley is a wheel, having a groove cut in its circumference, for the purpose of receiving a cord which passes over it. When motion is imparted to the cord, the pulley turns around its axis, which is generally supported by being attached to a beam above.

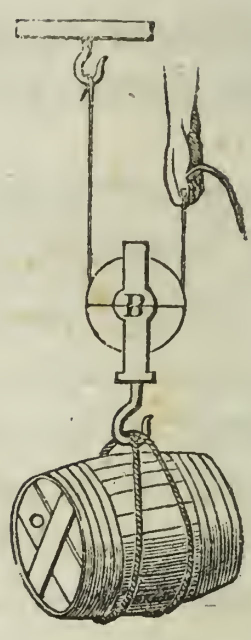


11. *How many kinds of pulleys are there?*

Pulleys are divided into two kinds, fixed pulleys and moveable pulleys.

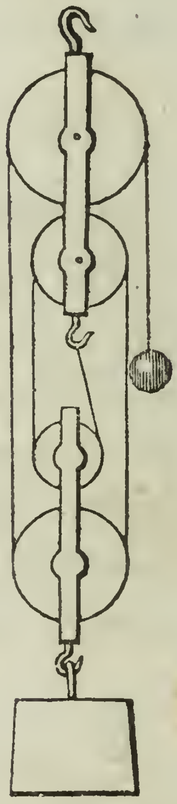
12. *Does a fixed pulley increase the power?*

When the pulley is fixed, it does not increase the power which is applied to raise the weight, but merely changes the direction in which it acts.



13. *Does a moveable pulley give any advantage in power?*

A moveable pulley gives a mechanical advantage. Thus, in the moveable pulley, the hand which sustains the cask does not actually support but one half the weight of it; the other half is supported by the hook to which the other end of the cord is attached.



14. *Will an advantage be gained by several moveable pulleys? What will be lost?*

If we have several moveable pulleys the advantage gained is still greater, and a very heavy weight may be raised by a small power. A longer time, however, will be required, than with a single pulley. It is indeed a general principle in machines, that *what is gained in power is lost in time*, and this is true for all machines.

15. *Is there an actual loss of power? What does it arise from?*

There is also an actual loss of power, viz., the resistance of the machine to motion, arising from the rubbing of the parts against each other, which is called the *friction* of the machine. This varies in the different machines, but must always be allowed for, in calculating the power

necessary to do a given work. It would be wrong, however, to suppose that the loss was equivalent to the gain, and that no advantage is derived from the mechanical powers. We are unable to augment our strength, but by the aid of science we so divide the resistance, that by a continued exertion of power we accomplish that which it would be impossible to effect by a single effort.

If in attaining this result we sacrifice time, we cannot but see that it is most advantageously exchanged for power.

16. *In the moveable pulley, what proportion exists between the power and the weight?*

It is plain that, in the moveable pulley, all the parts of the cord will be equally stretched, and hence, each cord running from pulley to pulley will bear an equal part of the weight; consequently, the *power will always be equal to the weight, divided by the number of cords which reach from pulley to pulley.*

EXAMPLES.

1. In a single immoveable pulley, what power will support a weight of 60 lbs.?

2. In a single moveable pulley, what power will support a weight of 80 lbs.?

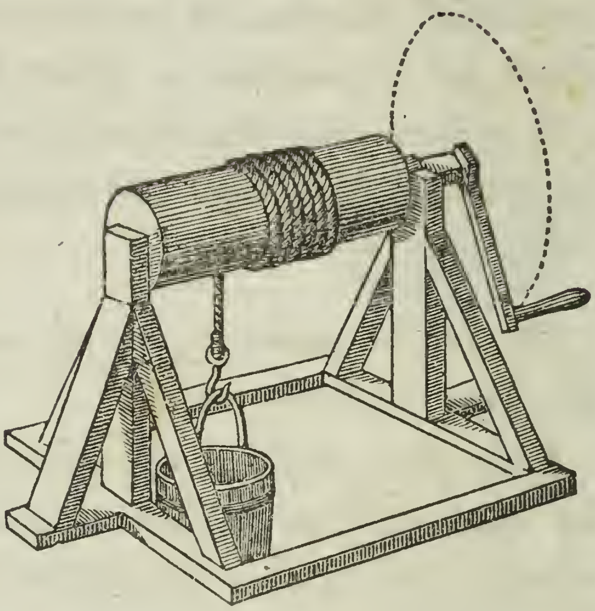
3. In two moveable pulleys with 5 cords, (see last fig.,) what power will support a weight of 100 lbs.?

Ans. 20 *lbs.*

WHEEL AND AXLE.

17. *Of what is the wheel and axle composed? How is the axle supported?*

This machine is composed of a wheel or crank, firmly attached to a cylindrical axle. The axle is supported at its ends by two pivots, which are of less diameter than the axle around which the rope is coiled, and which turn freely about the points of support.



18. *What is the proportion between the power and weight?*

In order to balance the weight, we must have,

The power to the weight, as the radius of the axle to the length of the crank, or radius of the wheel.

EXAMPLES.

1. What must be the length of a crank or radius of a wheel, in order that a power of 40 lbs. may balance a weight of 600 lbs., suspended from an axle of 6 inches radius?

Ans. $7\frac{1}{2}$ *ft.*

2. What must be the diameter of an axle, that a power of 100 lbs. applied at the circumference of a wheel of 6 feet diameter may balance 400 lbs.?

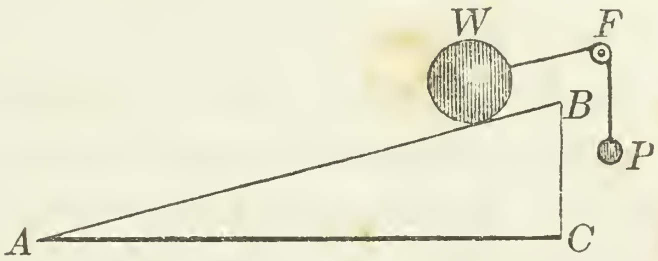
Ans. $1\frac{1}{2}$ *ft.*

INCLINED PLANE.

19. *What is an inclined plane?*

The inclined plane is nothing more than a slope or declivity, which is used for the purpose of raising weights. It is not difficult to see that a weight can be forced up an inclined plane more easily than it can be raised in a vertical line. But in this, as in the other machines, the advantage is obtained by a partial loss of power.

20. *What proportion exists between the power and the weight, when they are in equilibrium?*



If a weight W be supported on the inclined plane ABC by a cord passing over a pulley at F, and the cord from the pulley to the weight be parallel to the length of the plane AB, the power P will balance the weight W, when

$$P : W :: \text{height } BC : \text{length } AB.$$

It is evident that the power ought to be less than the weight, since a part of the weight is supported by the plane.

EXAMPLES.

1. The length of a plane is 30 feet, and its height 6 feet: what power will be necessary to balance a weight of 200 lbs.?

Ans. 40 *lbs.*

2. The height of a plane is 10 feet, and the length 20 feet: what weight will a power of 50 lbs. support?

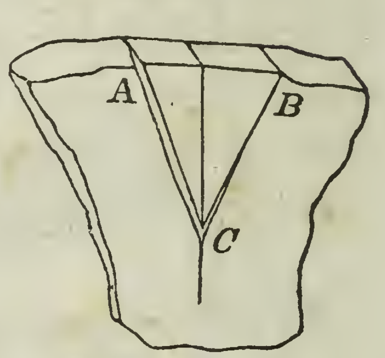
Ans. 100 *lbs.*

3. The height of a plane is 15 feet, and length 45 feet: what power will sustain a weight of 180 lbs.?

Ans. 60 *lbs.*

THE WEDGE.

21. *What is the wedge, and what is it used for*

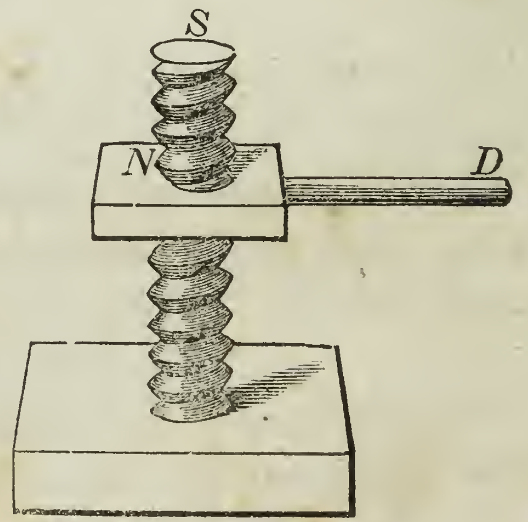


The wedge is composed of two inclined planes, united together along their bases, and forming a solid ACB. It is used to cleave masses of wood or stone. The resistance which it overcomes is the attraction of cohesion of the body which it is employed to separate. The wedge acts principally by being struck with a hammer or mallet on its head, and very little effect can be produced with it, by mere pressure.

All cutting instruments are constructed on the principle of the inclined plane or wedge. Such as have but one sloping edge, like the chisel, may be referred to the inclined plane; and such as have two, like the axe and the knife, to that of the wedge.

THE SCREW.

22. *Of how many parts is the screw composed? Describe its parts and uses.*



The screw is composed of two parts, the screw S, and the nut N.

The screw S is a cylinder with a spiral projection winding around it, called the *thread*. The nut N is perforated to admit the screw, and within it is a groove into which the thread of the screw fits closely.

The handle *D*, which projects from the nut, is a lever which works the nut upon the screw. The power of the screw depends on the distance between the threads. The closer the threads of the screw, the greater will be the power, but then the number of revolutions made by the handle *D* will also be proportionally increased; so that we return to the general principle—what is gained in power is lost in time. The power of the screw may also be increased by lengthening the lever attached to the nut.

The screw is used for compression, and to raise heavy weights. It is used in cider and wine presses, in coining, and for a variety of other purposes.

GENERAL REMARKS.

All machines are composed of one or more of the six machines which we have described. We should remember, that friction diminishes very considerably the power of machines.

There are no surfaces in nature which are perfectly smooth. Polished metals, although they appear smooth, are yet far from being so. If, therefore, the surfaces of two bodies come into contact, the projections of the one will fall into the hollow parts of the other, and occasion more or less resistance to motion. In proportion as the surfaces of bodies are polished, the friction is diminished, but it is always very considerable, and it is computed that it generally destroys one-third the power of the machine.

Oil or grease is generally used to lessen the friction. It fills up the cavities of the rubbing surfaces, and thus makes them slide more easily over each other.

SECTION IV.

OF SPECIFIC GRAVITY.

1. *What is the specific gravity of a body?*

The specific gravity of a body is the relation which the weight of a given magnitude of that body bears to the weight of an equal magnitude of a body of another kind.

2. *When is one body said to be specifically heavier than another?*

If two bodies are of the same bulk, the one which weighs the most is said to be specifically heavier than the other. On the contrary, one body is said to be specifically lighter than another, when a certain bulk or volume of it weighs less than an equal bulk of that other.

Thus, if we have two equal spheres, each one foot in diameter, the one of lead and the other of wood, the leaden one will be found to be heavier than the wooden one; and hence, its specific gravity is greater. On the contrary, the wooden sphere being lighter than the leaden one, its specific gravity is less.

3. *What does the greater specific gravity indicate? What is density?*

The greater specific gravity of a body indicates a greater quantity of matter in a given bulk, and consequently the matter must be more compact, or the particles nearer together. This closeness of the particles is called *density*. Hence, if two bodies are of equal bulk or volume, their weights or specific gravities will be proportional to their densities.

4. *If two bodies are of the same specific gravity, how will the weights be?*

If two bodies are of the same specific gravity, or density, their weights will be proportional to their bulks.

5. *If a body be immersed in a fluid, what will take place?*

If the body is specifically heavier than the fluid, it will sink on being immersed. It will, however, descend less rapidly through the fluid than through the air, and less power will be required to sustain the body in the fluid than out of it. Indeed, it will lose as much of its weight as is equal to the weight of a quantity of fluid of the same bulk. If the body is of the same specific gravity with the fluid, it loses all its weight, and requires no force but the fluid to sustain it. If it be lighter, it will be but partially immersed, and a part of the body will remain above the surface of the fluid.

6. *What do we conclude from the preceding article?*

1st. That when a heavy body is weighed in a fluid, its weight will express the difference between its true weight and that of an equal bulk of the fluid.

2d. If the body have the same specific gravity with the fluid, its weight will be nothing.

3d. If the body be lighter than the fluid, it will require a force equal to the difference between its own weight and that of an equal bulk of the fluid to keep it entirely immersed, that is, to overcome its tendency to rise.

7. *What is necessary in comparing the weights of bodies?*

In comparing the weights of bodies, it is necessary to take some one as a standard, with which to compare all others.

8. *What is generally taken as the standard?*

Rain-water is generally taken as this standard.

9. *What is the weight of a cubic foot of rain-water?*

A cubic foot of rain-water is found, by repeated experiments, to weigh $62\frac{1}{2}$ pounds, avoirdupois, or 1000 ounces. Now, since a cubic foot contains 1728 cubic inches, it follows that one cubic inch weighs .03616898148 of a pound. Therefore, if the specific gravity of any body be multiplied by .03616898148, the product will be the weight of a cubic inch of that body in pounds avoirdupois. And if this weight be then multiplied by 175, and the product divided by 144, the quotient will be the weight of a cubic inch in pounds troy; since 144 lbs. avoirdupois is just equal to 175 lbs. troy.

10. *How will the specific gravity of a body be to that of the fluid in which it is immersed?*

Since the specific gravities of bodies are as the weights of equal bulks, the specific gravity of a body will be to the specific gravity of a fluid in which it is immersed, as the true weight of the body to the weight lost in weighing it in the fluid. Hence, *the specific gravities of different fluids are to each other as the weights lost by the same solid immersed in them.*

11. *How do you find the specific gravity of a body, when the body is heavier than water?*

1st. Weigh the body first in air and then in rain-water, and take the difference of the weights, which is the weight lost.

2d. Then say, as the weight lost is to the true weight, so is the specific gravity of the water to the specific gravity of the body.

EXAMPLES.

1. A piece of platina weighs 70.5588 lbs. in the air, and in water only 66.9404 lbs.: what is its specific gravity, that of water being taken at 1000?

First, $70.5588 - 66.9404 = 3.6184$ lost in water.

Then, $3.6184 : 70.5588 :: 1000 : 19500$, which is the specific gravity, or weight of a cubic foot of platina.

2. A piece of stone weighs 10 lbs. in air, but in water only $6\frac{3}{4}$ lbs.: what is its specific gravity?

Ans. 3077.

12. *How do you find the specific gravity of a body when it is lighter than water?*

1st. Attach another body to it of such specific gravity, that both may sink in the water as a compound mass.

2d. Weigh the heavier body and the compound mass separately, both in water and in open air, and find how much each loses by being weighed in water.

3d. Then say, as the difference of these losses is to the weight of the lighter body in the air, so is the specific gravity of water to the specific gravity of the lighter body.

EXAMPLES.

1. A piece of elm weighs 15 lbs. in open air. A piece of copper which weighs 18 lbs. in air and 16 in water is attached to it, and the compound weighs 6 lbs. in water: what is the specific gravity of the elm?

Copper.	*Compound.*
18 in air.	33 in air.
16 in water.	6 in water.
2 loss.	27 loss.

Then, $27 - 2 = 25 =$ difference of losses.

Then, as 25 : 15 : : 1000 : 600, which is the specific gravity of the elm.

2. A piece of cork weighs 20 lbs. in air, and a piece of granite weighs 120 lbs. in air, and 80 lbs. in water. When the granite is attached to the cork the compound mass weighs $16\frac{3}{4}$ lbs. in water: what is the specific gravity of the cork?

Ans. 240.

13. *How do you find the specific gravity of fluids?*

1st. Weigh any body whose specific gravity is known, both in the open air, and in the fluid, and take the difference, which is the loss of weight.

2d. Then say, as the true weight is to the loss of weight, so is the specific gravity of the solid to the specific gravity of the fluid.

EXAMPLES.

1. A piece of iron weighs 298.1 ounces in the air, and 259.1 ounces in a fluid; the specific gravity of the iron is 7645: what is the specific gravity of the fluid?

First, 298.1 — 259.1 = 39 loss of weight:

Then, 298.1 : 39 :: 7645 : 1000, which is the specific gravity of the fluid: hence the fluid is water.

2. A piece of lignumvitæ weighs $42\frac{3}{4}$ ounces in a fluid, and $166\frac{5}{8}$ ounces out of it: what is the specific gravity of the fluid—that of lignumvitæ being 1333?

Ans. 991, which shows the fluid to be liquid turpentine or Burgundy wine.

Note.—In a similar manner the specific gravities of all liquids may be found from the following table.

TABLE OF SPECIFIC GRAVITIES.

	Sp. gr.	wt. cub. in.
Platina, hammered	20.336	11.777
Platina	19.500	11.285
Pure cast gold	19.258	11.145
Mercury	13.568	7.872
Cast lead	11.352	6.569
Pure cast silver	10.474	6.061
Cast copper	8.788	5.085
Cast brass	8.395	4.856
Hard steel	7.816	4.523
Cast cobalt	7.811	4.520
Cast nickel	7.807	4.513
Bar iron	7.788	4.507
Cast tin	7.291	4.219
Cast iron	7.207	4.165
Cast zinc	7.190	4.161
		wt. cub. ft. lbs.
Limestone	3.179	198.68
White glass	2.892	
Chalk	2.784	174.00
Marble	2.742	171.38
Alabaster	2.730	
Pearl	2.684	
Slate	2.672	167.00
Pebble	2.664	166.50
Green glass	2.642	
Flint and spar	2.594	162.12
Common stone	2.520	157.50
Paving stones	2.416	151.00
Sulphur	2.033	127.06
Brick	2.000	125.00
Ivory	1.822	
Bone of an ox	1.659	
Honey	1.456	
Lignumvitæ	1.333	83.31

	Sp. gr.	wt. cub. ft.
Ebony	1.331	83.18
Oak, 60 years old	1.170	73.12
Amber	1.078	
Beer	1.034	
Milk	1.030	
Sea water	1.028	
Distilled water	1.000	
Liquid turpentine	.991	
Burgundy wine	.991	
Camphor	.989	
Oak, English	.970	60.62
Bees' wax	.965	
Tallow	.945	
Olive oil	.915	
Logwood	.913	57.06
Box, French	.912	57.00
Wax	.897	
Oak, Canadian	.872	54.50
Alder	.800	50.00
Apple tree	.793	49.56
Ash and Dantzic oak	.760	47.50
Maple and Riga fir	.750	46.87
Cherry tree	.715	44.68
Beech	.696	43.50
Elder tree	.695	43.44
Walnut	.671	41.94
Pear tree	.661	41.31
Pitch pine	.660	41.25
Cedar	.596	37.25
Mahogany	.560	35.00
Elm and West India fir	.556	34.75
Larch	.544	34.00
Poplar	.383	23.94
Cork	.240	15.00
Air at the earth's surf.	$.001\frac{2}{7}$	

REMARK.—In the table of specific gravities, the cubic foot of water, which weighs 1000 ounces, is taken as the standard, and the figures in the column of specific gravity show how many times each substance is heavier or lighter than water. If the number opposite each substance be multiplied by 1000, the product will be the weight of a cubic foot of that substance, in ounces. The other column shows the weight in ounces of a cubic inch, or the weight in pounds of a cubic foot.

14. *How do you find the solidity of a body when its weight and specific gravity are given?*

As the tabular specific gravity of the body is to its weight in ounces avoirdupois, so is 1 cubic foot to the content in cubic feet.

EXAMPLES.

1. What is the solid content of a block of marble, that weighs 10 tons, its specific gravity being 2742?

First 10 tons = 358400 ounces.

Then 2742 : 358400 :: 1 : $130\frac{970}{1371}$, which is the content in cubic feet.

NOTE.—If the answer is to be found in cubic inches, multiply the ounces by 1728.

2. How many cubic inches in an irregular block of marble, which weighs 112 pounds, allowing its specific gravity to be 2520?

3. How many cubic inches of gunpowder are there in 1 pound weight, its specific gravity being 1745?

Ans. $15\frac{3}{4}$, *nearly.*

4. How many cubic feet are there in a ton weight of dry oak, its specific gravity being 925?

Ans. $38\frac{138}{185}$.

THE END.

May 4th 1891